Varieties whose generic affine normal space is normal at more than one point

Giacomo Maletto

KTH Royal Institute of Technology, Stockholm

gmaletto@kth.se

A fundamental concept in metric algebraic geometry is the affine normal space $N_X(x) := x + (T_xX)^\perp$ of a variety $X \subset \mathbb{R}^n$ at a smooth point $x \in X$: for example, x is a critical point of the distance function $d_u|_X$ from a data point $u \in \mathbb{R}^n$ if and only if $u \in N_X(x)$, and (x, y) is a bottleneck if and only if $y \in N_X(x)$ and $x \in N_X(y)$. We study the pathological case where the generic affine normal space is normal at more than one point on the variety. In such cases, every point becomes a bottleneck, and critical points of the distance function exhibit inherent relationships. Our analysis connects this phenomenon to the question of when distinct varieties yield the same family of affine normal spaces. We pay special attention to two settings: offset hypersurfaces and rotation-minimizing frames along space curves.