Morse theory of distance functions between algebraic varieties

Isaac Ren

KTH Royal Institute of Technology

isaacren@kth.se

We study the generalization of Morse theory to distance functions from an algebraic variety, restricted to another algebraic variety. In particular, we show that, generically, this distance function is "Morse" in the sense that it has a finite number critical points, and these critical points are all nondegenerate.

To make sense of this result, we first study non-smooth Lipschitz functions, which includes distance functions between subsets of Euclidean space. After defining appropriate notions of subdifferentials and critical points, we restrict our scope to continuous selections, as presented by Agrachev, Pallaschke, and Scholtes [1], where we have a notion of nondegenerate critical points. This leads to Morse-like results, which we present.

We then turn to the algebraic setting, where we compare our critical points with bottleneck and Euclidean distance degrees. We show, for generic complete intersections of degrees greater or equal to 4, that the critical points of the distance function from one surface to another are nondegenerate, and that their cardinality has a finite upper bound, which we make explicit.

This talk is partially based on joint work with Andrea Guidolin, Antonio Lerario, and Martina Scolamiero [2].

References

[1] A. A. Agrachev, D. Pallaschke, and S. Scholtes, On Morse theory for piecewise smooth functions, *J. Dynam. Control Systems*, **3**(4):449–469, 1997.

[2] A. Guidolin, A. Lerario, I. Ren, M. Scolamiero, Morse theory of Euclidean distance functions from algebraic hypersurfaces, arXiv:2402.08639.