The generalization of modern portfolio theory in the context of parametric polynomial systems

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Modern portfolio theory, or mean-variance analysis, is a Nobel Memorial Prize winner (economist Harry Markowitz, 1952) mathematical model for assembling a portfolio of assets such that the expected return is maximized while the level of risk is minimized. Classically the variance of the return of the portfolio is used as a measure of risk, because it is tractable when assets are combined into portfolios. The main idea of the original theory is to consider a linear combination of the expected value and the variance of the portfolio (this gives a quadratic function in the variables that are the weights of the given asset classes) called the utility function and compute its maxima. A typical utility function is the expected return minus half of the variance.

In this talk we generalize this idea and consider higher order cumulants as well in the utility function and study its critical points. So we consider

$$L(\alpha_1, \ldots, \alpha_n) = w_1\kappa_1(P) + \ldots + w_k\kappa_k(P),$$

where the portfolio is $P = \alpha_1 X_1 + \ldots \alpha_n X_n + (1 - \sum_{i=1}^{n} \alpha_i) X_{n+1}$, with random variables X_i being the assets and $w_1, \ldots w_k$ the unknown weights associated to the utility function L. We study the number of critical points as we vary both the degree of L and the w_i's, the different types of local optima, discriminantal loci of the w_i and other related concepts.