A family of network evolution models with moderate density

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We study a parametrized family of discrete time network evolution models. The evolution of the graph is based on constructions and deletions of k-cliques. Using martingale theory, we prove an almost sure limit theorem for the number of vertices, then show its asymptotic normality. We obtain an almost sure limit theorem for the degree of a fixed vertex. We also present an asymptotic normality result for the degree of a fixed vertex. A functional limit theorem is obtained for the number of vertices. A multidimensional functional limit theorem is proved for the joint behaviour of the degrees of several fixed vertices. For the proof, we apply martingale functional limit theorems. Besides mathematical proofs, we offer simulation results supporting our theorems.

References

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