Estimating the convex relaxation of the ideal magnetohydrodynamics equations

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In this talk we investigate the explicit convex relaxation of the ideal magnetohydrodynamics equations, which are given by the incompressible Euler equations with Lorentz force induced by a magnetic field, coupled with a Faraday-Maxwell induction equation and Ohm's law, to describe the evolution of the magnetic field. More precisely, the equations are

$$\begin{aligned} \partial_t u + \operatorname{div}\,(u \otimes u - B \otimes B) + \nabla p = 0, \\ \partial_t B + \operatorname{div}\,(B \otimes u - u \otimes B) = 0, \\ \operatorname{div}\, u = \operatorname{div}\, B = 0, \end{aligned}$$

for either $(x,t) \in \mathbb{R}^3 \times (0,T)$, or $(x,t) \in \mathbb{T}^3 \times (0,T)$ with appropriate zero-mean conditions for the fluid velocity $u(x,t) \in \mathbb{R}^3$ and the magnetic field $B(x,t) \in \mathbb{R}^3$. The fluid pressure is given by $p(x,t) \in \mathbb{R}$.

The MHD system is used as a model for electrically conducting fluids, such as for instance magnetic behaviour in plasmas and liquid metals.

Our goal is to provide a non-trivial lower estimate on the lamination hull and an upper estimate on the Λ-convex hull, the latter giving inequalities which will be satisfied by weak limits of weak solution of the ideal MHD equations, which serve as a model of averaged turbulent magnetohydrodynamical flows. The talk is based on the results obtained in [1].

References

[1] B. Fazekas, J. J. Kolumbán, Estimating the convex relaxation of the ideal magnetohydrodynamics equations, preprint 2025, arxiv.org/abs/2505.10230