Sharp Sobolev inequalities on Riemannian manifolds with nonnegative Ricci curvature

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In this talk we present sharp Sobolev inequalities on Riemannian manifolds with nonnegative Ricci curvature, where the asymptotic volume ratio plays a crucial role, being involved in the optimal Sobolev constants. In order to achieve our results, we exploit two approaches:

(a) Schwarz-type symmetrization combined with the sharp isoperimetric inequality, based on the distorted Brunn-Minkowski inequality;

(b) Optimal Mass Transport (OMT) combined with fine comparison and limiting arguments.

In particular, the latter approach answers an open question of Cordero-Erausquin, Nazaret and Villani (Adv. Math., 2004) on the applicability of the OMT to provide sharp Sobolev inequalities on noncompact Riemannian manifolds. The talk is mainly based on joint works with Zoltán M. Balogh.

References

[1] Balogh Z, Kristály A, Sharp isoperimetric and Sobolev inequalities in spaces with nonnegative Ricci curvature, *Math. Ann.* 385 (2023), no. 3-4, 1747–1773.

[2] Cordero-Erausquin D, Nazaret B, Villani C, A mass-transportation approach to sharp Sobolev and Gagliardo-Nirenberg inequalities. *Adv. Math.* 182(2) (2004), 307–332.

[3] Kristály A, Sharp Sobolev inequalities on noncompact Riemannian manifolds with $\mathrm{Ric} \geq 0$ via Optimal Transport theory, *Calc. Var.* (2024) 63:200.