Convergence rate for the longest at most T-contaminated runs of heads

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In this paper, we study the usual coin tossing experiment. Erdős and Rényi in their famous paper [3] proved results concerning the length of the longest pure head run. We call a head run containing T tails T-interrupted (in other words T-contaminated) head run. Erdős and Révész [4] obtained almost sure limit theorems for the length of the longest T-interrupted head run. Földes [5] presented asymptotic results for the distribution of the length of the longest T-interrupted head run applying Sevastyanov's Poisson limit theorem. Arratia, Gordon and Waterman [1] used Poisson approximation to prove accompanying distributions for the length of the longest T-interrupted head run.

However, our numerical studies show, that for $T > 0$, the rates of convergences are quite slow both in the result of Földes [5] and the result of Arratia, Gordon and Waterman [1]. In our paper, we present new accompanying distributions for the length of the longest at most T-interrupted head run. To obtain the convergence rate, we apply the Poisson approximation given in [1]. In the particular cases of $T = 1, 2$, an alternative proof can be given by using a lemma of Csáki, Földes and Komlós [2]. We shall apply that lemma for a certain problem of trinary experiment, too. We also give simulation results showing the performance of our theorems.

References

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