Sharp Hardy-type inequalities on special irrevesible Finsler manifolds

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The sharpness of Hardy-type inequalities is well-understood in the reversible Finsler setting, while infinite reversibility implies the failure of these functional inequalities. But what happens in the irreversible case, when the reversibility is finite? Here, the picture is much less clear, in fact, there are no concrete examples where Hardy-type inequalities remain sharp. In this talk we present two such examples involving two celebrated inequalities: first, the Hardy inequality (both in its classical and weighted form, assuming non-positive flag curvature), and second, the McKean-type spectral gap estimate (assuming strong negative flag curvature). In both cases, we construct families of Finsler metric measure manifolds where these inequalities turn out to be sharp. Our main technical tool is a Finslerian extension of the method of Riccati pairs—a technique originally developed for proving Hardy inequalities—which also guides the construction of the examples presented here.