Characteristic Polynomial of Tensors

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Eigenvectors and eigenvalues of matrices are classical objects of study with a wide range of applications. An important tool to determine the eigenpairs of a given matrix is to determine its characteristic polynomial, its roots correspond to the eigenvalues of the matrix.

Tensors are the natural generalization of matrices to higher dimensions, in fact, a matrix is an order two tensor. In recent years many extensions of the eigentheory of matrices to tensors have been proposed. One of such extensions is the notion of eigentensors, which provides a well-defined notion of eigenvalue, and so also the notion of characteristic polynomial of tensors. We will show that for tensors in $(\mathbb{C}^{n+1})^{\otimes d}$ there exists a variety of dimension at least n of tensors sharing the same (generic) characteristic polynomial, i.e., tensors that have the same eigenvalues. Moreover, we will show that there are only finitely many symmetric tensors (homogeneous polynomials) in $\mathcal{S}^d(\mathbb{C}^{n+1})$ that shares the same characteristic polynomial for $n = 1$ and $(n, d) = (2, 3)$. This is joint work with Francesco Galuppi, Fulvio Gesmundo and Lorenzo Venturello.