# Limit theorems for runs containing two types of contaminations

**István Fazekas, Borbála Fazekas, Michael Suja**

Department of Mathematics
University of Debrecen

fazekas.istvan@inf.unideb.hu
borbala.fazekas@science.unideb.hu
michael.suja@science.unideb.hu

The problem of the length of the longest head run for $n$ Bernoulli random variables was studied in the classical paper by Erdős and Rényi [1]. In this paper, we define and study the limiting distribution of the first hitting time and the accompanying distribution for the length of the longest at most 1 + 1 contaminated sequence of runs with trinary trials.

Let $X_1, X_2, \ldots, X_N$ be a sequence of independent random variables with three possible outcomes; 0, +1 and −1 labelled as success, failure of type I and failure of type II, respectively with the distribution

$$P(X_i = 0) = p, \quad P(X_i = +1) = q_1 \text{ and } P(X_i = -1) = q_2,$$

where $p + q_1 + q_2 = 1$ and $p > 0, \quad q_1 > 0, \quad q_2 > 0$.

An $m$ length section of the above sequence is called a pure run if it contains only 0 values. It is called a one-type contaminated run if it contains precisely one non-zero element either a +1 or a -1. On the other hand, it is called a two-type contaminated run if contains precisely one +1, and one -1 while the rest of the elements are 0's. A run is called at most 1 + 1 contaminated if it is either pure, or one-type contaminated, or two-type contaminated. So for an arbitrary fixed $m$, let $A_n = A_{n,m}$ denote the occurrence of the event at the $n^{th}$ step, that is, there is an at most 1 + 1 contaminated run in the sequence $X_n, X_{n+1}, \ldots X_{n+m-1}$ and $\bar{A}_n$.

**Theorem 1** *Let $\tau_m$ be the first hitting time of the at most 1 + 1 contaminated run of heads having length $m$. Then, for $x > 0$,*

$$P(\tau_m \alpha P(A_1) > x) \sim e^{-x} \tag{1}$$

*as $m \to \infty$. Here $P(A_1) = p^m + m(1-p)p^{m-1} + m(m-1)p^{m-2}q_1q_2$ and $\alpha$ is defined by the parameters of the process.*

Let $\mu(N)$ be the length of the longest at most two-type contaminated run in $X_1, X_2, \ldots, X_N$.

**Theorem 2** *For integer $k > 0$,*

$$P(\mu(N) - [m(N)] < k) = \exp\left(-p^{-\left(\log\left(C_0 p^{-2} q_1 q_2\right) + H(k - \{m(N)\})\right)}\right)\left(1 + \mathrm{O}\left(\frac{1}{(\log N)^3}\right)\right), \tag{2}$$

*where $m(N)$, $C_0$ and $H(x)$ are defined by the parameters of the process, $[m(N)]$ denotes the integer part of $m(N)$ and $\{m(N)\}$ denotes the fractional part of $m(N)$.*

The proofs of the above theorems depends on the fulfilment of some conditions given in the main Lemma of Csáki et al. [2].

## References

[1] Erdős, P.; Rényi, A. On a new law of large numbers, *J. Analyse Math.* **23** (1970), 103-111.

[2] Csáki, E.; Földes, A.; Komlós, J. Limit theorems for Erdős-Rényi type problems. *Studia Sci. Math. Hungar.* **22** (1987), 321–332.