A branching process based network evolution model describing N-interactions

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The aim of network theory is to describe real life networks like social networks, communication networks, trade networks, etc. A mathematical model for a network is a random graph. A pioneering paper in network theory is the paper [1] by Barabási and Albert.

In our paper, we present a new continuous time network evolution model driven by a multi-type branching process. We continue the lines of [4] and [2].

Now, we outline the structure of our model. The basic units are teams. Every team attracts new incomers. Teams are represented by cliques. The clique size can be $1, 2, \dots, N$, where N is a fixed integer. At the initial time $t = 0$, we start with a single team, it can be any n-clique, $1 \leq n \leq N$. It is called the ancestor. At certain random time a new member, i.e. a new node joins to the ancestor. So a new clique appears. Then the new clique also attracts a new member, that is a new node. So again a new clique appears and it starts its own reproduction process.

In out paper, after fixing the details of the evolution process, we obtain several limit theorems having the following shape.

Let n be fixed, $1 \leq n \leq N$. Let ${}_kT(t)$ denote the number of all n-cliques being born up to time t if the ancestor of the population was a k-clique, $k = 1, \dots, N$. Then

$$\lim_{t\to\infty} e^{-\alpha t}{}_kT(t) = {}_kW \frac{v_k u_n}{\alpha D(\alpha)}$$

almost surely for $k = 1, \dots, N$, where v_k, u_n, and $D(\alpha)$ are non-random and they are given by the parameters of the process, ${}_kW$ is an almost surely non-negative random variable, $E_kW = 1$, ${}_kW$ is a.s. positive on the event when the total number of offspring converges to infinity.

The proofs are based on known results of multi-type branching processes, see e.g. [3]. The mathematical theorems are supported by computer simulations, too.

References

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