

## Sharp spectral gaps and Hardy-Rellich inequalities via convexity-based techniques

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Exploiting the convexity of  $\xi \mapsto |\xi|^p$  we construct general functional inequalities on non-positivity curved Riemannian manifolds concerning the integral of an unknown function, its gradient, laplacian and radial gradient. Using them, we first provide sharp spectral gap estimates in terms of curvature bound and dimension for clamped plates (for  $p > 1$ ) and buckling problems (for  $p = 2$ ), respectively. Next we extend the classical and weighted Rellich inequality (for  $p > 1$ ) to non-positively curved manifolds. Finally we give short alternative proofs to a number of improved Rellich type inequalities (for  $p = 2$ ) on Euclidean and hyperbolic spaces, without spherical harmonics decompositions. Some higher-order inequalities are obtained as well.