A Generalized Least Action Principle for Rayleigh-Taylor subsolutions

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Minimizing the action leads to conserving the energy. Anyone with an undestanding of classical mechanics knows this. However, as seen from Onsager for instance, turbulent flows display anomalous energy dissipation. Hence, they can't satisfy a least action principle, at least not with the classical notion of action, i.e. kinetic energy minus potential energy. In our work we try to show that such flows can satisfy a least action principle if one modifies the potential energy, using a function that in a sense measures how far you are from not being turbulent at a given point. An appropriate differentiation of this "measure of turbulence" gives the measure of energy dissipation on the level of the Euler-Lagrange equation associated with the least action principle.

The main technical difficulty consists of showing sufficient regularity for the associated minimizer, such that one can obtain that it fits into our framework of convex integration subsolutions introduced in our previous papers modelling the Rayleigh-Taylor instability. For this we needed to push the limits of the state of the art in degenerate elliptic regularity theory, this is also where potential future improvements of our result could be hiding.

References

[1] B. Gebhard, J. Hirsch, J. J. Kolumbán, On a degenerate elliptic problem arising in the least action principle for Rayleigh-Taylor subsolutions, preprint 2022.