The latest applications of certain minimax theorems

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In this lecture, I will offer an overview of the latest applications of certain minimax theorems. Here are two samples.

THEOREM 1. - *Let* X *be a topological space,* E *a real normed space,* $S \subseteq E^*$ *a convex set weakly-star dense in* E^*, $I : X \to \mathbf{R}$, $\psi : X \to E$. *Assume the* $\psi(X)$ *is not convex and that, for every* $\eta \in S$, *the function* $I + \eta \circ \psi$ *is lower semicontinuous and inf-compact in* X.

Then, there exists $\tilde{\eta} \in S$ *such that the function* $I + \tilde{\eta} \circ \psi$ *has at least two global minima in* X.

THEOREM 2. - *Let* E *be a reflexive real Banach space and let* $C \subset E$ *be a closed convex set, with non-empty interior, whose boundary is sequentially weakly closed and non-convex.*

Then, for every function $\varphi : \partial C \to \mathbf{R}$ *and for every convex set* $S \subseteq E^*$ *dense in* E^*, *there exists* $\tilde{\gamma} \in S$ *having the following property: for every strictly convex lower semicontinuous function* $J : C \to \mathbf{R}$, *Gâteaux differentiable in* $\mathrm{int}(C)$, *such that* $J_{|\partial C} - \varphi$ *is constant in* ∂C *and* $\lim_{\|x\| \to +\infty} \frac{J(x)}{\|x\|} = +\infty$ *if* C *is unbounded,* $\tilde{\gamma}$ *is an algebraically interior point of* $J'(\mathrm{int}(C))$ *(with respect to* E^**).*