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Invited speakers

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Tibor Illés
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Zoltán Muzsnay
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Biagio Ricceri
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Automatic fake news detection

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The emergence and progression of social media have drastically changed the speed and extent of access to news, providing us with high-speed information acquisition, on the other hand, it is also possible to be flooded with a massive amount of information. This tremendous load of information greatly encumbers manually checking the legitimacy of every piece. In this presentation, we will try to tackle the fake news phenomenon from the perspective of machine learning, discussing, among other things, how to automatically detect fake news, how much accuracy can be achieved this way, and whether we can generate effective fake news. Furthermore, we will also discuss some of the challenges of automatic detection of fake news.

Sufficient linear complementarity problems: algorithms and computational results

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Linear complementarity problems (LCP) generalize some fundamental problems of mathematical optimization like linear programming (LP) problem, linearly constrained quadratic programming (LQP) problem and some other. It admits an enormous number of applications in economics, engineering, science, and many other fields. After all these, it is not surprising that LCPs are usually NP-complete problems (S.J. Chung, 1989). The three most significant classes of algorithms for solving LCP problems are: pivot algorithms (PA), interior point algorithms (IPA) and continuation methods. Because, both PA and IPA have been developed earlier for LP and LQP problems it is quite natural idea to test them on LCP problems, as well. Concept of sufficient matrices, as generalization of positive semidefinite matrices, has been introduced 30 years ago by Cottle et al. (1989). LCPs with sufficient matrices possess many important properties, like the solution set is convex and polyhedral; guarantees the finiteness of PAs and (pseudo) polynomial behaviour of the IPAs.

The largest matrix class where the interior point algorithms (IPA) are polynomial is the class of $P^*(\kappa)$-matrices, for given nonnegative real parameter κ. The union for all possible κ parameters of $P^*(\kappa)$-matrices forms the class of P^*-matrices. This class of matrices has been introduced by Kojima et al. in 1991. Known IPAs for LCPs with $P^*(\kappa)$-matrices under the interior point assumption, possess polynomial iteration complexity depending on the problem size n, parameter κ and the bit length L of the rational data of the LCP. After all of these, it is a natural question: What is the relation between the sufficient and P^*-matrices? Väliaho (1996) proved that the P^*-matrices are exactly those which are sufficient. Unfortunately, there are two important, negative results related to sufficient matrices. P. Tseng (2000) proved that decision problem related to the membership of matrices for P0- and column sufficient matrices are all co-NP-complete. While de Klerk and Eisenberg-Nagy showed that there exists such $P^*(\kappa)$-matrices for which the κ value is exponential in the size n of the problem. Furthermore, for sufficient LCPs, it is meaningful to introduce dual LCP problem and it can be proved that from sufficient primal- and dual LCP problem, exactly one has solution, that is an interesting, nice and (quite) new generalization of the old Farkas' lemma. There are still several open questions in the area of sufficient LCPs. More importantly, solution methods developed for sufficient LCPs helps us in trying to solve LCP problems with more general matrices.

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Tangent algebra of a diffeomorphism group and its application

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In this talk the notion of the tangent algebra of a (not necessarily smooth) subgroup of the diffeomorphism group $Diff(M)$ of a compact manifold M is introduced. We prove that this tangent algebra is a Lie subalgebra of the Lie algebra of smooth vector fields on M. The construction can be generalized to subgroups of any (finite or infinite dimensional) Lie groups. The tangent Lie algebra introduced this way is a generalization of the classical Lie algebra in the smooth case. As a working example, we discuss in detail the tangent structure of the holonomy group and the fibered holonomy group of Finsler manifolds.

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The latest applications of certain minimax theorems

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In this lecture, I will offer an overview of the latest applications of certain minimax theorems. Here are two samples.

Theorem 1. *Let X be a topological space, E a real normed space, $S \subseteq E^*$ a convex set weakly-star dense in E^*, $I : X \to \mathbf{R}$, $\psi : X \to E$. Assume the $\psi(X)$ is not convex and that, for every $\eta \in S$, the function $I + \eta \circ \psi$ is lower semicontinuous and inf-compact in X.*

Then, there exists $\tilde{\eta} \in S$ such that the function $I + \tilde{\eta} \circ \psi$ has at least two global minima in X.

Theorem 2. *Let E be a reflexive real Banach space and let $C \subset E$ be a closed convex set, with non-empty interior, whose boundary is sequentially weakly closed and non-convex.*

Then, for every function $\varphi : \partial C \to \mathbf{R}$ and for every convex set $S \subseteq E^*$ dense in E^*, there exists $\tilde{\gamma} \in S$ having the following property: for every strictly convex lower semicontinuous function $J : C \to \mathbf{R}$, Gâteaux differentiable in $\mathrm{int}(C)$, such that $J_{|\partial C} - \varphi$ is constant in ∂C and $\lim_{\|x\| \to +\infty} \frac{J(x)}{\|x\|} = +\infty$ if C is unbounded, $\tilde{\gamma}$ is an algebraically interior point of $J'(\mathrm{int}(C))$ (with respect to E^*).

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Characteristic Polynomial of Tensors

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Eigenvectors and eigenvalues of matrices are classical objects of study with a wide range of applications. An important tool to determine the eigenpairs of a given matrix is to determine its characteristic polynomial, its roots correspond to the eigenvalues of the matrix.

Tensors are the natural generalization of matrices to higher dimensions, in fact, a matrix is an order two tensor. In recent years many extensions of the eigentheory of matrices to tensors have been proposed. One of such extensions is the notion of eigentensors, which provides a well-defined notion of eigenvalue, and so also the notion of characteristic polynomial of tensors. We will show that for tensors in $(\mathbb{C}^{n+1})^{\otimes d}$ there exists a variety of dimension at least n of tensors sharing the same (generic) characteristic polynomial, i.e., tensors that have the same eigenvalues. Moreover, we will show that there are only finitely many symmetric tensors (homogeneous polynomials) in $\mathcal{S}^d(\mathbb{C}^{n+1})$ that shares the same characteristic polynomial for $n = 1$ and $(n, d) = (2, 3)$. This is joint work with Francesco Galuppi, Fulvio Gesmundo and Lorenzo Venturello.

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Contributed Talks

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ThesisSense: Using Large Language Models to extract meaningful information from theses

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In recent years, significant advancements in natural language processing have led to the development of large language models, exemplified by OpenAI's ChatGPT model. These models offer the capability to effectively extract diverse forms of information from textual documents, encompassing elements such as keywords and emotional tones. Leveraging the OpenAI API, textual data can be transformed into numerical representations, which, upon being stored within vector databases, facilitate the execution of semantic searches across a corpus of documents, thereby enabling interactions with our own textual materials.

This presentation elucidates a systematic approach for the extraction of valuable insights from academic theses. By deconstructing each document into discrete segments, the employed large language model is harnessed to derive keywords from the abstract sections. Subsequently, these keywords are transformed into a word cloud, affording a comprehensive snapshot of the theses encompassing a designated cohort. Furthermore, embeddings generated via OpenAI's technology are crafted for each individual document and then systematically organized within a localized vector database.

Upon projecting these embeddings onto a two-dimensional plane, discernible patterns emerge, visually portraying clusters of related documents. Documents expounding similar subject matters manifest as proximate points within this visual representation. Notably, the incorporation of semantic search functionalities within this document pool becomes feasible, thus enabling nuanced information retrieval from the aggregated body of documents.

Keywords: large language models, vector databases, embeddings, semantic search, information extraction

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Some uniqueness results for strongly singular problems

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We consider a strongly singular problem of the form

$$\begin{cases} -\Delta u &= \frac{f(u)}{u^{\delta}} \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \partial\Omega\,, \end{cases}$$

where Ω is a bounded smooth domain in $\mathbb{R}^N$, $\delta \geq 1$ and $f : [0,+\infty[\to]0,+\infty[$ is continuous with the property that the function $u \to \frac{f(u)}{u^{1+\delta}}$ is strictly decreasing in $]0,+\infty[$. With further restriction on either the behavior of f near the origin or on the range of δ, we prove that the problem admits at most one positive solution.

Based on [1].

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Solutions for nonlinear elliptic equations with singular potentials

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In this talk, we present recent results on the existence and classification of solutions for some nonlinear elliptic equations with singular potentials. This is joint work with Florica Cîrstea. The presentation is partially supported by CNCS-UEFISCDI Grant No. PN-III-P1-1.1-PD-2021-0037.

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Sharp spectral gaps and Hardy-Rellich inequalities via convexity-based techniques

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Exploiting the convexity of $\xi \mapsto |\xi|^p$ we construct general functional inequalities on non-positivity curved Riemannian manifolds concerning the integral of an unknown function, its gradient, laplacian and radial gradient. Using them, we first provide sharp spectral gap estimates in terms of curvature bound and dimension for clamped plates (for $p > 1$) and buckling problems (for $p = 2$), respectively. Next we extend the classical and weighted Rellich inequality (for $p > 1$) to non-positively curved manifolds. Finally we give short alternative proofs to a number of improved Rellich type inequalities (for $p = 2$) on Euclidean and hyperbolic spaces, without spherical harmonics decompositions. Some higher-order inequalities are obtained as well.

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A branching process based network evolution model describing N-interactions

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The aim of network theory is to describe real life networks like social networks, communication networks, trade networks, etc. A mathematical model for a network is a random graph. A pioneering paper in network theory is the paper [1] by Barabási and Albert.

In our paper, we present a new continuous time network evolution model driven by a multi-type branching process. We continue the lines of [4] and [2].

Now, we outline the structure of our model. The basic units are teams. Every team attracts new incomers. Teams are represented by cliques. The clique size can be $1, 2, \ldots, N$, where N is a fixed integer. At the initial time $t = 0$, we start with a single team, it can be any n-clique, $1 \leq n \leq N$. It is called the ancestor. At certain random time a new member, i.e. a new node joins to the ancestor. So a new clique appears. Then the new clique also attracts a new member, that is a new node. So again a new clique appears and it starts its own reproduction process.

In out paper, after fixing the details of the evolution process, we obtain several limit theorems having the following shape.

Let n be fixed, $1 \leq n \leq N$. Let ${}_kT(t)$ denote the number of all n-cliques being born up to time t if the ancestor of the population was a k-clique, $k = 1, \ldots, N$. Then

$$\lim_{t\to\infty} e^{-\alpha t}\,{}_kT(t) = {}_kW\frac{v_k u_n}{\alpha D(\alpha)}$$

almost surely for $k = 1, \ldots, N$, where v_k, u_n, and $D(\alpha)$ are non-random and they are given by the parameters of the process, ${}_kW$ is an almost surely non-negative random variable, $E_kW = 1$, ${}_kW$ is a.s. positive on the event when the total number of offspring converges to infinity.

The proofs are based on known results of multi-type branching processes, see e.g. [3]. The mathematical theorems are supported by computer simulations, too.

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Modified Deep Learning Models to Handle Noisy Labels

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Deep neural networks have excellent performance in image classifications tasks, but they are in need of large sets of training data with correct labels. This is a drawback, since labeling is either difficult or too expensive in many cases. The available datasets are often contaminated by label noise, that is why the challenge of learning with noisy labels has become an important research topic with several directions [3], [4]. Even though deep neural networks tend to learn the simple, consistent patterns first, they can easily overfit to noisy labels. If we are able to prevent this overfitting and treat the label noise during the training process, we can obtain models with better generalization ability.

To address this problem, several methods were proposed using a broad range of ideas from the field of machine learning, such as label refurbishment, bootstrapping, regularization or using a noise adaptation layer.

In our work, we have investigated the possibilities of the improvement of a recent technique in the topic of learning with label noise. We have applied some modifications to various points of the training process, evaluated those adjusted models and drawn conclusions from the results.

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The spherical curvature of arbitrary algebraic varieties

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In this talk, based on [1], we will study the complexity involved in the computation of the critical spherical curvature points of an arbitrary algebraic variety. We present properties of the critical spherical curvature points as well as an algorithm for computing them. We will see that critical spherical curvature is an algebraic number in the case of algebraic varieties. Finally, we prove that all singular points of the generalized evolute correspond to points in the critical curvature pairs variety. This generalizes classical findings.

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Subclasses of univalent mappings in $\mathbb{C}^n$ and the Graham-Kohr extension operator

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Let $\mathbb{B}^n \subseteq \mathbb{C}^n$ be the Euclidean unit ball in $\mathbb{C}^n$. We present two subclasses of convex, respectively starlike univalent mappings on $\mathbb{B}^n$ together with some interesting properties of them. Using the Graham-Kohr extension operator (introduced by I. Graham and G. Kohr in 2002) we study how the previous subclasses are preserved from one to several complex variables.

In the final part, we present some numerical examples that indicate a positive answer to a question proposed in the paper: the extension of the class K (of convex functions on the unit disc) through the Graham-Kohr extension operator.

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Dynamics of Excess Mortality

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Excess mortality during a pandemic reflects a confluence of factors beyond just direct disease fatalities. It includes indirect deaths from overwhelmed healthcare, postponed medical procedures, medication shortages, skipped immunizations, mental health crises leading to substance abuse or suicides, reluctance to seek care due to infection concerns, economic-driven malnutrition, inactivity from lockdowns, increased domestic violence, overlooked tropical diseases, and stretched emergency services. In this talk we employ statistical modeling to explore how various national vaccination campaigns and quality of life and prosperity indicators impacted these outcomes amid the COVID-19 crisis.

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National exam results in Romania - behind the scenes

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The 8th grade national assessment and baccalaureate exam results are of paramount importance not only for students and teachers but also for educational institutions alike. These data greatly influence the future possibilities of students, whether it's about career opportunities or further education, and they also reflect the quality of education in schools. We implemented an automated system that aims to collect, clean, and visualize these exam results of Romanian students between 2015 and 2023 for the end users. Visualizing these results provides an opportunity for effective evaluation of education since parents and teachers can gain a comprehensive understanding of students' knowledge and progress based on the data. This can help improve the learning process and the development of teaching programs.

The main goal was not only to enable easy and fast access to the data, more accurate inference drawing, and better understanding of the result for everyone but also to compare these exam results by nationality, highlighting its weaknesses. Sapviz.ro is the only website that provides help with exams in hungarian and contains reports on the results of hungarian students.

Keywords: data extraction, data cleaning, data visualization, information extraction

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[2] http://bacalaureat.edu.ro/

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Order in disorder with examples from graph theory

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In randomly created structures (whether natural or artificial) very often there exist ordered substructures. Here we discuss some such graph-theoretic structures:

- the existence of a Hamiltonian path in a graph obtained by arbitrarily directing the edges of a complete graph,
- Ramsey type extremal graph problems,
- Turán type extremal graph problems,
- a conjecture on pairwise arc-independent Hamiltonian circuits in De Bruijn graphs.

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Transitioning the AlgoRythmics project from the dance floor to the stage

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The AlgoRythmics project, an undertaking spanning a twenty-year period, has embarked upon a noteworthy juncture wherein it converges the realms of computer science and the performing arts. The project has garnered widespread international acclaim for its dance choreography illustrations, which offer compelling visual expositions of foundational computer science algorithms, with a particular emphasis on algorithms related to search and sorting strategies [1, 2, 3]. Building upon the foundation of this achievement, the research collective has recently undergone a transition from the milieu of dance floors to that of theatrical stages, introducing an inventive modality to elucidate intricate computer science algorithms, encompassing methodologies such as greedy algorithms and dynamic programming, through the medium of captivating short films.

The created performance encapsulates a thematic ethos reminiscent of escape rooms, thereby imbuing the visualization of algorithms with an element of captivating engagement. This transformative enterprise is characterized by a synergy with a proficient choreographer hailing from a University of Arts, in tandem with an adept cinematographer, thereby effectuating a seamless transmutation of painstakingly devised algorithmic scenarios into enchanting theatrical performances. By virtue of its collaborative efforts with art students affiliated with the University of Arts, the AlgoRythmics project assumes a pioneering role in promulgating an interdisciplinary confluence of computer science and the performing arts, thereby pushing the frontiers of collaborative endeavors and ensnaring the collective attention of audiences in the process.

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A Generalized Least Action Principle for Rayleigh-Taylor subsolutions

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Minimizing the action leads to conserving the energy. Anyone with an undestanding of classical mechanics knows this. However, as seen from Onsager for instance, turbulent flows display anomalous energy dissipation. Hence, they can't satisfy a least action principle, at least not with the classical notion of action, i.e. kinetic energy minus potential energy. In our work we try to show that such flows can satisfy a least action principle if one modifies the potential energy, using a function that in a sense measures how far you are from not being turbulent at a given point. An appropriate differentiation of this "measure of turbulence" gives the measure of energy dissipation on the level of the Euler-Lagrange equation associated with the least action principle.

The main technical difficulty consists of showing sufficient regularity for the associated minimizer, such that one can obtain that it fits into our framework of convex integration subsolutions introduced in our previous papers modelling the Rayleigh-Taylor instability. For this we needed to push the limits of the state of the art in degenerate elliptic regularity theory, this is also where potential future improvements of our result could be hiding.

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Clamped plates on curved spaces

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The talk deals with the clamped plate problem, initially formulated by Lord Rayleigh in 1877, and solved by M. Ashbaugh & R. Benguria (Duke Math. J., 1995) and N. Nadirashvili (Arch. Ration. Mech. Anal., 1995) in 2 and 3 dimensional euclidean spaces. We consider the same problem on both negatively and positively curved spaces, and provide various answers depending on the curvature, dimension and the size of the clamped domain.

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An application of Shepard operator in image reconstruction

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Shepard's method, introduced in [5], is one of the best ways to solve scattered data approximation problems, i.e., to reconstruct an unknown functions' values from some given scattered data. Multiple improvements were made during the time to obtain better results than the ones provided by the original method (e.g., [1], [3]). One example of improvement is the combination of the bivariate Shepard operator with inverse quadratic and inverse multiquadric radial basis functions which was introduced in [2].

Besides function approximation, another topic that uses Shepard method is the one of image reconstruction. With some ideas from [4] and [6], we will focus on reconstructing damaged black and white or color images, using the combined Shepard operator of inverse quadratic and inverse multiquadric type and evaluate the results by studying the approximation errors.

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Security problems in smart greenhouses

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Due to the expanding population, climate change, and the rising demand for resources, agriculture is encountering escalating challenges. The implementation of advanced technologies in agriculture therefore plays a particularly important role. Intelligent greenhouses assume a pivotal role in the efficient production of high-quality agricultural products. Research into smart greenhouses farming has been underway for a considerable duration. However, the quantity of studies that identify risks and methodically present security issues remains relatively limited. In this presentation, we will delve into these security problems.

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Nonsmooth elliptic problems on Riemannian manifolds

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We investigate a large class of elliptic differential inclusions on non-compact complete Riemannian manifolds which involves the Laplace-Beltrami operator and a Hardy-type singular term. Depending on the behavior of the nonlinear term and on the curvature of the Riemannian manifold, we guarantee non-existence and existence/multiplicity of solutions for the studied differential inclusion. The proofs are based on nonsmooth variational analysis as well as isometric actions and fine eigenvalue properties on Riemannian manifolds. The results are also new in the smooth setting.

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Evolving teaching methods: balancing traditional teaching and e-learning in mathematics education

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Technology's fast evolution impacts teaching methods, especially due to the pandemic's shift to online education. Institutions seek a balance between traditional and e-learning, leveraging benefits from each. Conventional teaching encourages active participation, while online lectures rely on prepared slides and recordings.

The research explores students' views on two math education approaches: retaining traditional elements in online settings and carrying online practices back to physical classrooms. Data from sixty undergraduates, collected trough questionnaires and analyzed statistically, shows appreciation for both methods in term of satisfaction and perceived learning outcomes.

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An Adaptive Parameter Setting Technique for the Possibilistic Fuzzy c-Means Clustering Algorithm

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C-means clustering algorithms employing fuzzy partitions have two basic approaches. Chronologically the first, introduced by Dunn [1] and generalized by Bezdek [2] uses a probalilistic partition, and is called the fuzzy c-means (FCM) clustering model. Alternatively, the so-called possibilistic c-means (PCM) algorithm, introduced by Krishnapuram and Keller [3], relaxed the probabilistic constraint of the partition matrix and uses fuzzy membership functions that describe the compatibility of data vectors with the c clusters. The irony in the terminology is that PCM is more fuzzy than FCM.

Since both basic approaches have shortcomings in their behavior, mixed clustering methods were proposed, which were desingned to attenuate the unwanted phenomena. The most well-known mixed c-means clustering algorithm is the possibilistic-fuzzy c-means model proposed by Pal et al. [4], which combines the probabilistic and possibilistic components of the partition matrix as a linear combination.

This paper presents an alternative formulation of the PFCM clustering model, which allows for the adaptive tuning of the so-called possibilistic penalty terms of the algorithm. This way the user needs to set a reduced number of parameters, which makes the algorithm easier to handle. The proposed solution is inspired by the cluster size controlling version of the FCM algorithm proposed by Miyamoto and Kurosawa [5], and the self-tuning version of the PCM algorithm proposed by Szilágyi et al [6]. Numerical tests revealed that the proposed adaptive PFCM algorithm produces slightly better clusteing results in terms of partition quality indicators like cluster purity, normalized mutual information, and adjusted Rand index.

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Finding Nash-Stampacchia equilibrium points of Hirschleifer games using numerical algorithms

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This paper discusses the problem of finding Nash-Stampacchia equilibrium points of Hirschleifer games using numerical algorithms such as Quasi-Newton method and Nelder Mead algorithm. Nash equilibrium points are a subset of Nash-Stampacchia equilibrium points, which can be obtained as solutions of variational inequalities. These inequalities correspond to critical points of the payoff functions. Various numerical algorithms have been studied to find these critical points. The Nelder Mead algorithm is found to be one of the best options because it does not require gradients which makes it suitable to study on hyperbolic spaces as well. This study provides a detailed analysis of the results. We also present the Nelder Mead algorithm on Hadamard manifolds, more precisely on Poincaré modell.

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See, Learn, Code: Elevating Data Structures and Algorithms with Videos

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Responding to the changing landscape of computer science education, this study introduces a novel method that utilizes instructional videos to enhance understanding and practical application of data structures and algorithms, crucial components of problem-solving and algorithmic thinking. This pedagogical strategy seamlessly integrates instructional videos into a university-level course dedicated to data structures and algorithms.

The approach involves well-crafted videos that simplify complex concepts through visual examples, interactive explanations, and real-world demonstrations. These videos bridge the gap between abstract theories and real-world applications: the "See" element showcases algorithms in action, "Learn" offers engaging demonstrations, and "Code" encourages active participation, blending theoretical comprehension with hands-on practice.

Key to this method's effectiveness is the versatility of instructional videos. Going beyond traditional teaching, students can revisit, slow down, or accelerate learning as needed. This adaptability is especially valuable for mastering intricate concepts like data structures and algorithms, which often require repeated exposure. By enabling the review of challenging segments and accommodating individual learning paces, this approach fosters deeper understanding. It also benefits advanced learners who can move quickly, and the on-demand accessibility of videos seamlessly integrates learning into students' routines, promoting continuous and lasting comprehension.

This innovative educational approach not only meets the evolving demands of computer science education but also equips students with a flexible tool for lifelong learning and mastery.

Keywords: algorithms, data structures, visual learning, hands-on practice, multimedia learning

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Numerical methods for parameter estimation of random coefficient demand systems

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The BLP estimator (Berry, Levinsohn, and Pakes, 1995) [1] is widely used to estimate consumer preferences from a discrete-choice demand model with random coefficients. Essentially, this model is treated as a non-linear, constrained optimization problem where a non-linear system must be solved for every parameter value during the optimization process. Recently, other methods such as MPEC (mathematical programming with equilibrium constraints) [2] or ABLP (approximated BLP) [3] have been proposed. In this study, we analyze the nested fixed point approach with different components such as derivative and derivative-free outer algorithms, the recently proposed Spectral and Squarem [4] non-linear solvers for the inner loop. We also compare our approach with the MPEC and ABLP methods.

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Hardy-Brezis-Marcus type inequalities in Minkowski spaces.

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We present Hardy-Brezis-Marcus type inequalities on domains in Minkowski spaces and the impact of the mean curvature of the domain in such inequalities.

Some applications are also discused.

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Computing a Common Prior

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Morris (1994) and later Feinberg (2000) showed that a finite type space (information structure) attains a common prior if and only if there is no agreeable bet in it.

We also consider finite type spaces and observe that deciding about the existence of a common prior is equivalent with considering the intersection of affine spaces each is spanned by the types of a player. This observation implies that we can apply the Fredholm alternative (Fredholm, 1903), and conclude that the computational complexity of computing a common prior or an agreeable bet is strongly polynomial.

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Gender and Educational Background as Determinants of Computational Thinking Skills among First-Year Students at the Sapientia Hungarian University of Transylvania

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Computer skills and computational thinking are to a large extent socially biased. The research question addressed in this paper is whether men and women have different computational skills. The rationale for the study is provided by the lower share of female students in computer sciences which originates in the alleged gender differences in computational thinking. A randomized controlled trial was conducted with 228 first-year students from different specializations in September 2022. Three visualization methods were used: animation, folk dance and theatre performance. After having seen the visualizations, participants answered the same questions about the algorithms by filling in questionnaires.

This study presents the impact of the three visualization methods upon the understanding of algorithms in university students, as well as the gender differences in efficient learning with the different visualization methods.

Keywords: computational thinking, visualisation, gender, randomized controlled trial

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Limit theorems for runs containing two types of contaminations

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The problem of the length of the longest head run for n Bernoulli random variables was studied in the classical paper by Erdős and Rényi [1]. In this paper, we define and study the limiting distribution of the first hitting time and the accompanying distribution for the length of the longest at most `1 + 1` contaminated sequence of runs with trinary trials.

Let $X_1, X_2, \ldots, X_N$ be a sequence of independent random variables with three possible outcomes; 0, $+1$ and -1 labelled as success, failure of type I and failure of type II, respectively with the distribution

$$P(X_i = 0) = p, \quad P(X_i = +1) = q_1 \text{ and } P(X_i = -1) = q_2,$$

where $p + q_1 + q_2 = 1$ and $p > 0, \quad q_1 > 0, \quad q_2 > 0$.

An m length section of the above sequence is called a pure run if it contains only 0 values. It is called a one-type contaminated run if it contains precisely one non-zero element either a `+1` or a `-1`. On the other hand, it is called a two-type contaminated run if contains precisely one `+1`, and one `-1` while the rest of the elements are 0's. A run is called at most `1 + 1` contaminated if it is either pure, or one-type contaminated, or two-type contaminated. So for an arbitrary fixed m, let $A_n = A_{n,m}$ denote the occurrence of the event at the n^{th} step, that is, there is an at most `1 + 1` contaminated run in the sequence $X_n, X_{n+1}, \ldots X_{n+m-1}$ and $\bar{A}_n$.

Theorem 3. *Let τ_m be the first hitting time of the at most 1 + 1 contaminated run of heads having length m. Then, for $x > 0$,*

$$P(\tau_m \alpha P(A_1) > x) \sim e^{-x} \tag{1}$$

as $m \to \infty$. Here $P(A_1) = p^m + m(1-p)p^{m-1} + m(m-1)p^{m-2}q_1q_2$ and α is defined by the parameters of the process.

Let $\mu(N)$ be the length of the longest at most two-type contaminated run in $X_1, X_2, \ldots, X_N$.

Theorem 4. *For integer $k > 0$,*

$$P(\mu(N) - [m(N)] < k) = \exp\left(-p^{-\left(\log\left(C_0 p^{-2} q_1 q_2\right) + H(k - \{m(N)\})\right)}\right)\left(1 + \mathrm{O}\left(\frac{1}{(\log N)^3}\right)\right), \tag{2}$$

where $m(N)$, C_0 and $H(x)$ are defined by the parameters of the process, $[m(N)]$ denotes the integer part of $m(N)$ and $\{m(N)\}$ denotes the fractional part of $m(N)$.

The proofs of the above theorems depends on the fulfilment of some conditions given in the main Lemma of Csáki et al. [2].

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A new segmented and annotated colposcopy image set, image mapping necessary for the conization of the cervix

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It is well known that cervical cancer is the fourth most common cancer among women, and the proper diagnosis of this disease still remains a difficult challenge around the world. Considering the latter, colposcopy images are crucial in the diagnosis of cervical cancer. Given the fact that recognition of the cancerous lesions is not a simple task, there are some attempts for the development of malignant lesion recognition software based on machine learning algorithms. It must be taken into account that colposcopy image data sets are usually small with restricted amount of valuable data leading to a limited training capacity. In our paper we present a new colposcopy image set that can be used for cervical mapping. At the same time the segmentation and annotation of these images were made in order to prepare data for machine learning algorithms. In addition, this paper discusses some image enhancement techniques as well. As the proper analysis of colposcopy images of cervical malignant lesions depends primarily on the quality of the images taken, this topic is even more important when several doctors are discussing the diagnosis of a patient within the framework of telemedicine.

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On Sendov's conjecture

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Let $\mathbb{D} = \{z \in \mathbb{C} : |z| \leq 1\}$ be the closed unit disk in $\mathbb{C}$. Let $\mathbb{C}[z]$ denote the set of polynomials $P(z) = a_0 z^n + a_1 z^{n-1} + a_2 z^{n-3} + a_{n-1} z + a_n$, where $a_k \in \mathbb{C}$, $k \in \{0, 1, 2, \ldots, n\}$ and $n \in \mathbb{N}^*$.
We will study sufficient conditions regarding the roots of a polynomial $P \in \mathbb{C}[z]$ which imply the following conjecture, attributed to the bulgarian mathematician Blagovest Sendov.

Conjecture 1. *If all the roots of a polynomial $P \in \mathbb{C}[z]$ lie in $\mathbb{D}$ and z^* is an arbitrary root of the polynomial P then the disk $\{z \in \mathbb{C} : |z - z^*| \leq 1\}$ contains at least one root of P'.*

We will give sufficient conditions which imply the Sendov's conjecture.