Remote motion planning of rigid bodies immersed in a 2D perfect incompressible fluid

Olivier Glass **József J. Kolumbán** **Franck Sueur**

Paris Dauphine University - CEREMADE Leipzig University - Mathematics Institute
Bordeaux University - Mathematics Institute

Jozsef.Kolumban@math.uni-leipzig.de

We consider the motion of several rigid bodies immersed in a two-dimensional incompressible perfect fluid. The motion of the rigid bodies is given by the Newton laws with forces due to the fluid pressure and the fluid motion is described by the incompressible Euler equations. Our analysis covers the case where the circulations of the fluid velocity around the bodies are nonzero and where the fluid vorticity is bounded.

The whole system occupies a bounded simply connected domain with an external fixed boundary which is impermeable except on an open non-empty part where one allows some fluid to go in and out the domain by controlling the normal velocity and the entering vorticity. We prove that it is possible to exactly achieve any non-colliding smooth motion of the rigid bodies by the remote action of a controlled normal velocity on the outer boundary which depends on the state of the fluid-rigid bodies system, with zero entering vorticity. This extends the result of [1] where the exact controllability of a single rigid body immersed in a 2D irrotational perfect incompressible fluid from an initial position and velocity to a final position and velocity was investigated.

The proof relies on a nonlinear method to solve linear perturbations of nonlinear equations associated with a quadratic operator having a non-zero non-degenerate critical point. Here this method is applied to a quadratic equation satisfied by a class of boundary controls, which is obtained by extending the reformulation of the Newton equations performed in the uncontrolled case in [2] to the case where a control acts on the external boundary. The quadratic operator mentioned above is then obtained as a part of the force terms on the moving rigid bodies due to the fluid motion driven by the control on the fixed external boundary. A class of controls for which this operator has a non-zero non-degenerate critical point is constructed by complex analysis methods.

References

[1] Glass, O., Kolumbán, J. J., Sueur, F. (2020). External boundary control of the motion of a rigid body immersed in a perfect two-dimensional fluid. Analysis & PDE 13-3, 651–684.

[2] Glass, O., Lacave, C., Munnier, A., Sueur, F. (2019). Dynamics of rigid bodies in a two dimensional incompressible perfect fluid. Journal of Differential Equations, 267(6), 3561–3577.