On some particular tame Hall polynomials

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Classical Hall algebras associated with discrete valuation rings were introduced by Steinitz and Hall to provide an algebraic approach to the classical combinatorics of partitions. The multiplication is given by Hall polynomials which play an important role in the representation theory of the symmetric groups and the general linear groups. In 1990 Ringel defined Hall algebras for a large class of rings, namely finitary rings, including in particular path algebras of quivers over finite fields. Far reaching analogues of the classical ones, these Ringel-Hall algebras provided a new approach to the study of quantum groups using the representation theory of finite dimensional algebras. They can also be used successfully in the theory of cluster algebras or to investigate the structure of the module category.

In case of Ringel-Hall algebras corresponding to Dynkin quivers and tame quivers we know due to Ringel and Hubery, that the structure constants of the multiplication are again polynomials in the number of elements of the base field. These are the generalized Hall polynomials. If we are looking at Hall polynomials associated to indecomposable modules, the classical ones are just 0 or 1, the generalized ones in the Dynkin case are also known and have degree up to 5, however we do not have too much information about the generalized ones in the tame case.

Let k be an arbitrary field and Q a tame quiver of type $\tilde{D}_4$. Consider the path algebra kQ and the category of finite dimensional right modules mod-kQ.

We describe tools and techniques used to determine all the Hall polynomials F_{xy}^z associated to indecomposable modules $x, y, z \in$ mod-kQ. As a result we obtain a first list of tame Hall polynomials involving indecomposables of absolute defect different from 0 or 1.