Galois Connections in Subgroup Lattices

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Given two partially ordered sets $(P, \leq)$ and $(Q, \sqsubseteq)$, a pair of mappings $f : P \to Q$ and $g : Q \to P$ is called a *Galois connection* (or Galois correspondence) if the mappings satisfy the two conditions $x \leq (g \circ f)x$ and $(f \circ g)y \sqsubseteq y$ for any pair of elements x, y in P and Q, respectively. An equivalent characterization states that the inverse image under f of every principal down-set $\{y' \in Q | y' \sqsubseteq y\}$ of Q is a principal down-set of P, and this correspondence determines g uniquely. Therefore, the classification of Galois connections is closely related to the structure of the underlying lattices. We review some known facts about subgroup lattices $L(G)$ of p-primary groups G from the perspective of Galois connections $f, g : L(G) \to L(G)$ and point to interesting related problems.

References

[1] T. S. Blyth, *Lattices and Ordered Algebraic Structures*, Springer, 2005.