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## Abstracts

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#### Invited speakers

Pietro D'Avenia (Politecnico di Bari, Italia)

Krisztián Buza (Sapientia Hungarian University of Transylvania, Romania)
Cristian Cazacu (University of Bucharest & Gheorghe Mihoc–Caius Iacob Institute of Mathematical Statistics and Applied Mathematics of the Romanian Academy, Romania)
Alpár Richárd Mészáros (Durham University, UK)

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## Ground states for an Hartree-Fock type system

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Joint work in collaboration with Liliane Maia and Gaetano Siciliano.

We introduce an Hartree-Fock type system made by two Schrödinger equations in presence of a Coulomb interacting term and a *cooperative* pure power and subcritical nonlinearity depending on a parameter  $\beta \geq 0$ .

We present some results about the existence of radial ground states solutions and their *semitriviality* or *vectoriality* covering the whole range  $\beta \geq 0$ .

## **Operator Splitting and Alternating Process**

#### Lívia Boda

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Operator splitting is a widely and successfully used method in numerical analysis. It helps us when we have a very complicated Cauchy-problem, which we want to analyse. By using operator splitting, we get a series of easier Cauchy-problems which are linked through their initial conditions. By applying this method it is significantly easier to solve the problem of finding the numerical solution of the original problem.

The two most popular splitting methods are the sequential splitting (it is a first order method) and the Strang-Marchuk splitting method (it is a second order method). In this talk we analyse the relationship between these two methods, and analyse the following main question: how can we get from first order splitting methods to second order splitting methods. The answer is what we call "alternating" splitting. Furthermore, in the case of second-order methods, we also use the alternating splitting and analyse it.

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## Efficient program representation for code comprehension

#### Tibor Brunner, Zoltán Porkoláb

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In industrial environments where a legacy code base consists of multiple million lines of code, it is a challenging task to get a proper overview of the software. Code comprehension tools aim to support this process by providing fast navigation opportunities and visualizations like inheritance diagrams, function call chain detection, pointer analysis, etc. CodeCompass is a code comprehension framework that gathers statically collected information through extensible plug-ins like language parsers, metrics measurements or version control systems.

The challenging task of such a system is to store this huge amount of data in a database as concise as possible. It is a primary target to get fast access to this data in order to provide an efficient service for an extensive overview of the whole legacy system. In this paper we describe the data model that CodeCompass uses to store this wide range of information.

#### Why Romania is a much better place than you think

#### Krisztián Buza

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In his highly influential book [1], Hans Rosling et. al. describe that most of us have a systematically incorrect view of the situation of the World. This is primarily due to our outdated stereotypes. One of Rosling's key messages is that we need a data-driven or fact-based approach to understand our World, or a part of it. In this talk, we will review the book and apply a similar methodology to understand the situation of Romania. As a result, we will see that according to several essential indicators, Romania is close to rich and healthy countries of Western Europe and North America. While we do not claim that Romania is a perfect country, inline with the aforementioned book, we point out that things may be BAD and BETTER at the same time: compared with the ideal situation, most of the things are bad, however, they are much better than a few decades ago. In other words: "the world has improved and it's far from good" [2]. Despite the fact that the presentation is based on publicly available statistics, the interpretation of the numbers may be somewhat subjective. Therefore, an additional goal of this talk is to motivate fact-based discussions about what exactly could and should be improved and how.

- Hans Rosling et al. (2018) "Factfulness: Ten Reasons We're Wrong about the World -And Why Things Are Better Than You Think"
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## First and second order Caffarelli-Kohn-Nirenberg type inequalities: sharp constants and minimizers.

#### Cristian Cazacu

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We discuss sharp first and second order inequalities of Caffarelli-Kohn-Nirenberg (CKN) type in the euclidian space  $\mathbb{R}^N$ , where N denotes the dimension.

Firstly, we provide very short and self-contained proofs of a refined version of the main results by F. Catrina and D. Costa (J. Differential Equations 2009). Our results are sharp and minimizers are obtained in suitable functional spaces.

Secondly, we analyze second order CKN inequalities. This is equivalent to the study of uncertainty principles for special classes of vector fields. In particular, we show that when switching from scalar fields  $u : \mathbb{R}^N \to \mathbb{C}$  to vector fields of the form  $\vec{u} := \nabla U$ (U being a scalar field) the best constant in the Heisenberg Uncertainty Principle (HUP) increases from  $\frac{N^2}{4}$  to  $\frac{(N+2)^2}{4}$ , and the optimal constant in the Hydrogen Uncertainty Principle (HyUP) improves from  $\frac{(N-1)^2}{4}$  to  $\frac{(N+1)^2}{4}$ . We also provide minimizers for the improved sharp constants. As a consequence of our results we answer to an open question of Maz'ya (Integral Equations Operator Theory 2018, Section 3.9) in the case N = 2 regarding the HUP for divergence free vector fields.

This exposure is based on joint works with Joshua Flynn (University of Connecticut, USA) and Nguyen Lam (Memorial University of Newfoundland, Canada).

This talk is partially supported by CNCS-UEFISCDI Romania, Grant No. PN-III-P1-1.1-TE-2019-0456.

## IRP Classification and Probe Positioning Failure Detection in High-Resolution Esophageal Manometry using Machine Learning

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#### Abstract

High-Resolution Esophageal Manometry (HRM) is the most used technique for the study of Esophageal Motility Disorders (EMD) like Achalasia type I, type II, type III [1], Jackhammer Esophagus [2], etc. The HRM catheter is inserted transnasally and positioned to cover the full length of the esophagus during the procedure. It uses as many as 36 circumferential sensors and generates heat-map-like images, representing the muscle movement in the esophagus during a swallow. These sensors transmits intraluminal pressure data through a high-resolution catheter, which is then translated into dynamic esophageal pressure topography maps.

Based on these heat-map-like images the Chicago Classification (version 3.0) [3] is applied, which is a formal analytic framework for esophageal motility disorders used to make an esophageal motility diagnosis. The diagnosis of the EDM is a time consuming manual process, doctors needs to make different measurements, like measuring the Integrated Relaxation Pressure (IRP) [5], the Distal Contractile Integral (DCI) [4], etc. on the raw HRM images and then manually apply the Chicago Classification algorithm in order to determine the exact class or type of disorder. Furthermore the esophageal motility diagnosis can be highly affected by the positioning of the catheter, which is the reason why it is critical to make sure that the positioning of the catheter is precise and there is no place for misinterpretation in the generated esophageal pressure topography maps.

In this paper we present a Machine Learning based solution for detecting probe positioning failures in HRM images, which can be used before applying the Chicago Classification algorithm, this way maximizing the precision of the esophageal motility diagnosis. Furthermore, we created a classifier to automatically determine whether the Integrated Relaxation Pressure is in the normal range or it is higher than the cut-off, based solely on the raw pressure topography images. Determining the IRP type is one of the most important step in the Chicago Classification algorithm, so this work is the first step towards automating the Chicago Classification algorithm using Machine Learning techniques. Automating this algorithm will highly reduce the costs for the hospitals, because the esophageal motility diagnosis will be automatically done, requiring only a nurse to position the catheter, so no specialist will be needed anymore.

In order to detect probe positioning failures and to classify the IRP, firstly we preprocessed the raw HRM images. The first step in preprocessing of the images was to find the region of interest, by finding the part of the image which represents the exact moment of swallowing. After this step we resized the images and we rescaled the pixel values so they can be used as input for Deep Learning models. After preprocessing the images we tried multiple Deep Learning models to classify the images as normal or abnormal images (abnormal means wrong catheter position) and to determine the exact class of the IRP. The result of the trained Neural Networks (NN) were very promising, with an accuracy and f1-score over 95% for most of the NN models that we tried. The results of these experiments are presented at the end of our article.

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## Essential coalitions for non-balanced games

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Huberman (1980) introduces the notion of essential coalitions. In balanced games essential coalitions have the property of beeing only needed in the computation of the nucleolus of the game. In our paper we provide two generalizations of Huberman's result. Both generalizations give classes of coalitions which are only needed for computing the prenucleolus of an arbitrary TU-game. We also demonstrate that both generalizations are real generalizations of the class of essential coalitions by Huberman (1980), and that the two introduced classes of coalitions are not related to each other.

### Targeted static fault localization in Erlang programs

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Static source code analysis techniques may help the programmers in various tasks: code comprehension, testing, debugging, etc. They often need to reproduce executions that result in faulty behaviour. Program analysis techniques with symbolic execution can help to solve this task. In this paper we propose a method to select an appropriate execution path from the static control-flow graph that may lead to a given runtime error in Erlang software. We build our tool on the RefactorErl static analyser framework.

Fault localization is the act of identifying the locations of faults in a program. Even when bugs in software are discovered due to some faulty behavior (e.g. a runtime error occurs), finding the location of the fault is a non-trivial task. Error detection mechanisms are vital for building highly reliable systems. Fault localization is one of the most time consuming, and expensive part of software development and maintenance. Given the size and complexity of large-scale software systems today, manual fault localization becomes more and more futile, so effective automatic methods are needed.

In a concrete execution, a program is evaluated on a specific input and a single controlflow path is explored. Symbolic execution [1, 2] uses unknown symbolic variables in evaluation, allowing to simultaneously explore multiple paths that a program could take under different inputs. We can use symbolic execution to help us in fault localization. We target to find an execution path in the program, the "error path", that may result in a runtime error in a given point of the program. Thus we build a direct symbolic execution engine for a given execution path in the Erlang programs based on the RefactorErl framework [3]. We are using the SMT solver of Z3 [4] to solve the constraints that we gather during our analysis.

We can use symbolic execution to help us in fault localization. We target to find an execution path in the program, the "error path", that may result in a runtime error in a given point of the program. Thus we build a direct symbolic execution engine for a given execution path in the Erlang programs based on the RefactorErl framework [3]. We are using the SMT solver of Z3 [4] to solve the constraints that we gather during our analysis.

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## Operator splitting and its applications

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In the modelling of complex time-depending physical phenomena the simultaneous effect of several different sub-processes has to be described. The operators describing the sub-processes are as a rule simpler than the whole spatial differential operator. Operator splitting is a widely used procedure in numerical solution of such problems. The point in operator splitting is the replacement of the original model with one in which appropriately chosen groups of the sub-processes, described by the model, take place successively in time. This de-coupling procedure allows us to solve a few simpler problems instead of the whole one.

In the talk several splitting methods will be constructed (sequential splitting, Strang splitting, weighted splitting, additiv splitting, iterated splitting). We discuss the accuracy (local splitting error) of the methods. The stability and the convergence will be also discussed. We also examine the effect of the choice of the numerical method chosen to the numerical solution of the sub-problems in the splitting procedure. We list the main benefits and drawbacks of this approach.

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# Sharp existence and classification results for nonlinear elliptic equations in $\mathbb{R}^N \setminus \{0\}$ with Hardy potential

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We reveal the structure and asymptotic behavior near zero and infinity of all positive solutions for the nonlinear elliptic equation with Hardy potential  $(\star) -\Delta u - \frac{\lambda}{|x|^2}u + |x|^{\theta}u^q = 0$  in  $\mathbb{R}^N \setminus \{0\}$   $(N \geq 3)$ , where q > 1,  $\theta \in \mathbb{R}$  and  $\lambda \in \mathbb{R}$  are arbitrary. We provide the sharp range of the parameters such that there exist positive solutions of  $(\star)$  in  $\mathbb{R}^N \setminus \{0\}$ . We show that equation  $(\star)$  has either a unique solution or infinitely many solutions or no positive solutions. This is joint work with Florica Cîrstea. This presentation is partially supported by CNCS-UEFISCDI Grant No. PN-III-P1-1.1-TE-2019-0456.

## User authentication using deep learning on SapiMouse dataset

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In the last few years, there was an increase in applications that utilize user authentication and bot detection based on mouse movements. This led to increasing interest and curiosity regarding the analysis of human-computer interaction, and the topic of this paper is a specific area of this, namely mouse dynamics. In this talk, we introduce a new dataset called SapiMouse, which is valuable for training and evaluating neural networks, to be later applied in user authentication or bot detection systems. Firstly, we present the exploratory data analysis of this dataset, along with the tools and protocols we implemented for collecting the raw data. Further on, we discuss the user authentication results in light of the above mentioned dataset. To learn the features from the raw data, the system laid out here uses a convolutional neural network, instead of regular handcrafted features. The system achieved a 0.94 AUC performance, using merely 15 seconds of data.

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## Influencing factors in developers' code comprehension

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Long-term projects will inevitably face the problem of changing team members - especially in large development teams - and the neglect of certain components of the source code. Such projects tend to grow to a size where it becomes nearly impossible to know and understand the entire project. It is crucial to keep track of who is competent in each unit of the code in order to avoid information loss.

The contributions made by developers to the source code offer an obvious starting point when we try to map and quantify the software related knowledge of each developer. However, the contributions are diverse: apart from the quantity and significance of modifications, other factors also play an important role in code comprehension and software related knowledge, such as code reviews, pull request reviews, knowledge in documentation, or forgetting. In this talk, I present the different results derived from 3 important factors in code comprehension: the quantity and significance of modifications in the source code, and the inevitable forgetting.

## Double phase problems involving critical Sobolev nonlinearities

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In this talk, we discuss about recent results for double phase problems involving critical Sobolev nonlinearities. More precisely, our problems are driven by the so-called double phase operator given by

div  $(|\nabla u|^{p-2}u + a(x)|\nabla u|^{q-2}u)$  for  $u \in W^{1,\mathcal{H}}(\Omega)$ ,

set on an appropriate Musielak-Orlicz Sobolev space  $W^{1,\mathcal{H}}(\Omega)$ , with  $1 and <math>a \in L^{\infty}(\Omega)$  such that  $a(x) \geq 0$  a.e. in  $\Omega$ . Our problems present some difficulties due to the presence of singularities and the lack of compactness of the critical Sobolev embeddings for  $W^{1,\mathcal{H}}(\Omega)$ . Under suitable assumptions for weight a, exponents p and q, we are able to provide the existence and multiplicity of solutions for our problems, by applying different variational approaches. The results presented in this talk are based on the papers [1, 2, 3].

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## Label noise handling on MNIST with neural networks<sup>1</sup>

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In recent years, deep neural networks have reached very impressive performance in the task of image classification. However, these models require very large datasets with labeled training examples, and such datasets are not always available. The labeling process is often very expensive, or it is very difficult even for experts in a particular field. That is what can lead to the use of databases with label noise, which contain incorrectly labeled instances. Therefore, it is important to examine training on this kind of datasets. According to a widely accepted assumption, deep networks learn consistent, simple patterns in the beginning, and then it is followed by the learning of the harder examples with possibly incorrect labels. So treating the label noise in the train set can lead to a better generalization ability instead of overfitting to the wrong examples.

In this work, we investigate the possibilities of improving a classifier (which is an ensemble of deep neural networks) by handling the label noise in the training dataset. We classify with an ensemble of convolutional neural networks (CNNs). At the start, we train that ensemble on the original training dataset. Then we are going to apply a label noise cleansing technique on that data. Finally, we take a CNN ensemble with the same structure as our original CNN ensemble, and train it on the new dataset gained after treating the label noise. We evaluate and compare the performance of the ensemble classifiers and draw conclusions. Our goal is to study label correcting neural networks for preprocessing purpose. Preprocessing can be either relabeling or deleting items detected to have noisy labels. After preprocessing, usual CNNs are applied for the data. With preprocessing, the performance of very accurate convolutional networks can be further improved.

We conduct experiments on the MNIST dataset [2], which contains handwritten digits. It consists of images with  $28 \times 28$  grayscale pixels. The size of the training set is 60 000 examples and the test set has 10 000 samples. However, it contains some misleading items. We shall consider these misleading instances as inaccurately labeled ones so we can apply some known methods elaborated to handle noisy labels.

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<sup>&</sup>lt;sup>1</sup>This work was supported by the construction EFOP-3.6.3-VEKOP-16-2017-00002. The project was supported by the European Union, co-financed by the European Social Fund.

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## Nonlinearity of the non-Abelian gauge field theory on lattice considering the spectrum of Kolmogorov-Sinai entropy and complexity

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The Yang-Mills fields have an important role in the non-Abelian gauge field theory which describes the properties of the quark-gluon plasma. The real time evolution of the classical fields is given by the equations of motion which are derived from the Hamiltonians to contain the term of the SU(2) gauge field tensor[1]. This system shows chaotic behaviour[2, 5, 6]. The homogeneous Yang-Mills contains the quadratic part of the gauge field strength tensor  $F^a_{\mu\nu}$  in the Minkowski space, it is expressed by gauge fields  $A^a_{\mu}$ :

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu, \tag{1}$$

where  $\mu, \nu = 0, 1, 2, 3$  are space-time coordinates, the symmetry generators are labeled by a, b, c = 1, 2, 3, g is the bare gauge coupling constant and  $f^{abc}$  are the structure constant of the continuous Lie group. The generators of the Lie group fulfills the following relationship  $[T^b, T^c] = i f^{bcd} T^d$ .

The equation of motion can be expressed by covariant derivative in the adjoin representation:

$$\partial^{\mu}F^{a}_{\mu\nu} + gf^{abc}A^{b\mu}F^{c}_{\mu\nu} = 0.$$

The real time evolution of the classical Yang-Mills fields equations is derived by the Hamiltonian for SU(2) gauge field tensor to constraint the total energy on constant values and it satisfies the Gauss law[?, ?]. The microcanonical equations of motion are solved on  $N^d$  lattice and chaotic dynamics is studied by the time dependent entropy-energy relation, which was presented by the spectrum of Kolmogorov-Sinai entropy and the complexity[7].

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## Necessary Optimality Conditions for Generalized Weak Optimal Solutions in Set-Valued Optimization

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The specialized literature abounds in approaches of tackling the notion of an optimal solution for set-valued optimization problems, of the form:

$$\begin{cases} \min F(x) \\ G(x) \in C \end{cases}$$

wehere  $F: X \Rightarrow Y$  and  $G: X \Rightarrow Z$  are set-valued functions. Among first such attempts we mention the so-called scalar approach, where a certian poin  $x_0 \in X$  is said to be an optimal solution, if there exists  $y_0 \in F(x_0)$  such that  $y_0$  is a minimal point of the set  $\bigcup_{x \in XG(x) \in C} F(x)$ . Another approach, the so-called *the natural criteria* in set-valued optimization was first introduced by D. Kuroiwa in 1998 and it basically considers  $x_0 \in X$ as an optimal solution, if the set  $F(x_0)$  is minimal, when compared the other sets F(x), in terms of a set-inclusion relation, defined by the means of a convex cone.

Continuing Kuroiwa's idea, we introduce a new weak set-relation, with the help of the notion of the quasi-relative interior of a convex cone. In terms of that, we are able to deliver a new optimality notion of a weak solution of a set-valued optimization problems. This notion is accompanied by necessary optimality conditions. As well, we introduce a set-valued dual and a duality theorem. I began this path of research within [2] and [3]

Let A and B belong to  $\mathcal{P}_0(Y)$  and  $K \subseteq Y$  be a nonempty, pointed convex cone. Then we write:

- (a)  $A \leq_{\operatorname{ciK}}^{l} B$  if  $B \subseteq A + \operatorname{qiK}$  and  $A \leq_{\operatorname{ciK}}^{u} B$  if  $A \subseteq B \operatorname{qiK}$ .
- (b) an  $l Min_{qi}$ -efficient set of  $\mathcal{S}$ , if for each set  $B \in \mathcal{S}$  satisfying

 $B \leq_{aiK}^{l} A$ , the relation  $A \leq_{aiK}^{l} B$  holds.

(c) an  $u - Min_{qi}$ -efficient set of  $\mathcal{S}$ , if for each set  $B \in \mathcal{S}$  satisfying

 $B \leq^{u}_{qiK} A$ , the relation  $A \leq^{u}_{qiK} B$  holds.

An extension of the classical Fenchel theorem can be stated in this case, as it is seen below: let  $x_0, x_1 \in \text{domF}$ , and let  $T \in \mathcal{L}(X, Y)$  be such that  $F(x_1) - Tx_1 \in -F^*_{\text{qi}_K}(T)$ . Then

- (a) If  $F(x_0) Tx_0 \leq_{\mathrm{div}}^l F(x_1) Tx_1$ , then  $F(x_1) Tx_1 \leq_{\mathrm{div}}^l F(x_0) Tx_0$ .
- (b) If  $F(x_0) Tx_0 \leq_{\text{div}}^l F(x_1) Tx_1$ , then  $F(x_1) Tx_1 \sim^l F(x_0) Tx_0$ .

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## Classes of holomorphic functions whose derivatives have positive real part

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Starting from paper [4] written by T.H. MacGregor we discuss about normalized holomorphic functions whose derivatives have positive real part. For the class

$$R = \left\{ f \in S : \operatorname{Re}[f'(z)] > 0, z \in U \right\}$$

we present a general distortion result (some upper bounds for the modulus of the k-th derivative of a function). We also prove some remarks on the functions whose derivatives have positive real part of order  $\alpha \in [0, 1)$ . We denote this class with

$$R(\alpha) = \left\{ f \in S : \operatorname{Re}[f'(z)] > \alpha, z \in U \right\}.$$

More details about holomorphic functions and classes of functions whose derivatives have positive real part can be found in [1, Chapter 2], [3], [4], [5, Chapter 4] and [6].

Based on paper [2] in the last part of the presentation we discuss about two new subclasses of normalized holomorphic functions whose derivatives have positive real part which generalize the classes R and  $R(\alpha)$ . For these subclasses (denoted here  $R_p$  and  $R_p(\alpha)$ ) we present some particular results and examples.

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## An Approach for Formalizing the Memory Consumption of C++ Standard Template Library Containers

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The Standard Template Library [1] (STL) which initially developed by Hewlett-Packard Company in 1994 became the library for the standard C++ language. In the following years, this was significantly expanded and enriched with many new elements in the C++11 / C++14 standards. The template library supports generic programming with containers, iterators (generalized pointers), and algorithms [2]. The template solution allows us to use the classes and functions with a given name for (almost) any type, according to the needs of the program. Containers have different characteristics in many ways:

- time required to insert or delete a new item
- access time to stored items
- regarding the memory usage, containers can be divided into two main groups: memory-contiguous and node-based

In STL, there is an asymptotic run-time guarantee for most library algorithm operations that are performed on containers, but there is less conversation about how container's memory is allocated or re-allocated on the heap. After all, a vector and a linked list have a different memory representation, such as a double-ended queue or a map. With n number of elements, we can be sure that these data structures will need different amounts of memory allocation even if they are of the same type of elements.

It is almost impossible to make an exact prediction because the size of each type is not known in advance, especially the user-defined types. However, it is possible to determine memory requirements in advance, such as that the size of an x container containing integers of n elements is sure to always be less than or equal to that of a y container, where x and y are different types of containers. In order to provide an even more generic solution, we disregard the platform-dependent memory allocation of C++ primitive types.

The purpose of this paper is to provide a detailed explanation of these guarantees. We present an approach how to formalize the heap memory requirements for certain containers.

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## The critical curvature degree of an algebraic variety

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This topic is about the complexity involved in the computation of the reach in arbitrary dimensions and in particular the computation of the critical spherical curvature points of an arbitrary algebraic variety in arbitrary dimension. We present properties of the critical spherical curvature points as well as an algorithm for computing them.

More information can be found in [1]

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## Long time behavior of the solutions for some diffusion processes on a metric graph

#### Liviu Ignat

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In this talk we present some recent result about the long time behavior of the solutions for some diffusion processes on a metric graph.

We study local and nonlocal (convolution type problems with an integrable kernel) evolution problems on a metric connected finite graph in which some of the edges have infinity length. We show that the asymptotic behaviour of the solutions is given by the solution of the heat equation, but on a star shaped graph in which there is only one node and as many infinite edges as in the original graph. In this way we obtain that the compact component that consists in all the vertices and all the edges of finite length can be reduced to a single point when looking at the asymptotic behaviour of the solutions. We prove that when time is large the solution behaves like a gaussian profile on the infinite edges.

This is a joint work with Julio D. Rossi (Buenos Aires) and Angel San Antolin (Alicante).

## Predicting at-risk students using weekly activities and Assessments

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#### Introduction

Higher education institutes face challenges caused by low course enrolments and beyond lower course completion rates. The issue higher dropping out rates is fast becoming a priority, and universities are seeking for strategies to improve students' retention rates. OCED reported that in Australia just 31% of students' completed a 4-year degree programme, US had 49% completions while UK is on top with 71% completions [1]. Lower retention rates are a serious threat to universities long term financial security. Hence, Universities are focusing on identifying strategies which ensure students successes and that can provide proactive actions to support students in their course work. Having some analytical strategy that can enable predictions on students' performances will help those institutes to make timely interventions for improving students' performance.

The Common use of tools like Student Management Systems (SMS) and Learning Management Systems (LMS) have supported higher education institutes in providing seamless online communication, in delivery of learning and teaching resources, designing interactive learning activities and managing academic assessments. In addition, they provide them with large datasets that are related to students' demographics student academic records and log files. These logs are based upon students' interactions with the LMS and have offered us with new research directions that can help in improving students' academic performance [2][3].

In the research, OU Learning Analytics Dataset (OULAD). The dataset comprises student demographics, clickstream history, and assessment submission information of 32,593 students over a course duration of 9 months, from 2014 to 2015. The data is composed of several courses, with each course being taught at different intervals in a year. Four distinct performance classes were defined: distinction, pass, fail, and withdrawal. A course belonging to Engineering and technology category was chosen with 1303 students. The OULAD comprised students' information regarding their interaction with the Virtual Learning Environment (VLE)—their assessments, quizzes, and course performances. VLE interaction was classified into 20 different activity types with each activity referring to a specific action, such as downloading or viewing lectures, course content, or quizzes. The current course has 536,837 records in the VLE log, and 10373 record in the assessments log. The research has three questions: does aggregating students weekly clicks and per week assessment when available predict at-risk student early? Other question was: does accumulating previous weeks assessments provide better prediction results compared to previous question results? Third question was: Which demographic features are affective in first weeks, before assessments availability, beside weekly activities?

#### Methodology and challenges

OULAD data is provided as tables, so it can be used directly with machine learning models. Logs were maintained daily; a "data" column represents a day - 0 to the end of the course. First step was to aggregate the events into weekly format. Then students VLE table and assessments were processed and divided into 34 sub datasets; a week dataset will have all the student who had an activity during that week. For first question, we followed this strategy:  $w_n = \sum_{i=0}^n W^i$  to obtain the dataset for week number n. Predictor variables for a given week is the count of total online activities performed that week. Second variable could be assessment scores (if accessible at that time). Second question datasets were accumulating previous assessments in the subsequent weeks as predictors in addition to the available assessment score at that week.

To answer question three: the first 3 weeks subsets predictors further analysed to select the best features by Information Gain.

To classify students in two group (either at-risk or not-at-risk of failing the course), those machine learning algorithms were used: Random Forest classifier (RF), Naive Bayes (NB), Logistic regression (LR), Linear Discriminating analysis (LDA). 10-fold cross validation was used to train all the classifiers, and F-measure was used as an evaluation metric.

#### Results

#### Strategy 1:

Results showed that LDA outperformed other models with score of 0.69 in first weekusing just clickstream data. In second week, LR was the best scoring 0.715, and LDA was close. RF was leading in third week with 0.777 score. All models scored improved 4-5% in this week as the first assessment grade was available. All models scores were 74% above starting from forth week.

#### Strategy 2:

The first 3 weeks data was similar to strategy 1 because during those weeks, only engagement data was available. In forth week and onward, the improvement was 3% and more in the models performances. The most noticeable improvement was in RF; Rf scored 0.692 in 4th week in strategy 1, while it scored 0.79 I in 4th week subset of strategy 2.

For answering research question 3, we applied feature importance and ranking techniques on the first three weeks datasets after combining demographic features with and engagement data. The purpose was to select just the affective features that would improve the performance.

To sum up, this research proposed two analytics strategies to predict at-risk students based on their weekly activities. Experiments shows that strategy two is better and was able to predict struggling students in early weeks. Also, feature selection techniques improved the prediction based on behaviour and demographic features.

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## Operator-based iterative methods for nonlinear elliptic PDEs

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Nonlinear elliptic problems arise in various physical and other applications where the model describes a stationary state of the given process, such as flow problems, elasticity or glaciology etc. The widespread way to solve such problems is a finite element discretization plus some Newton-like iteration. The talk summarizes earlier and recent work on an operator approach, where quasi-Newton methods are constructed via spectral equivalence, also interpreted as variable preconditioning. The considered problem classes depend on the structural conditions of the nonlinearity. The robustness of the method is illustrated by numerical tests for some real-life models.

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## EdTech and online teaching during the COVID-19 pandemic

#### Zoltán Kátai, David Iclanzan

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Online courses can often increase student's sense of isolation, impacting their ability to concentrate and learn [1]. With diminished social cues, communication becomes more "task-oriented, cold, and less personal than during face-to-face communication" [2, p. 461]. Consequently, several works studying online education and its outcomes highlight the key role the "strong teacher presence" plays in engaging learners in meaningful learning experiences [3, 4, 5, 6].

In this talk, we present our efforts and experiences in developing EdTech solutions that could lead to increased teacher presence and detail and analyze the gathered student feedback.

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## Remote motion planning of rigid bodies immersed in a 2D perfect incompressible fluid

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We consider the motion of several rigid bodies immersed in a two-dimensional incompressible perfect fluid. The motion of the rigid bodies is given by the Newton laws with forces due to the fluid pressure and the fluid motion is described by the incompressible Euler equations. Our analysis covers the case where the circulations of the fluid velocity around the bodies are nonzero and where the fluid vorticity is bounded.

The whole system occupies a bounded simply connected domain with an external fixed boundary which is impermeable except on an open non-empty part where one allows some fluid to go in and out the domain by controlling the normal velocity and the entering vorticity. We prove that it is possible to exactly achieve any non-colliding smooth motion of the rigid bodies by the remote action of a controlled normal velocity on the outer boundary which depends on the state of the fluid-rigid bodies system, with zero entering vorticity. This extends the result of [1] where the exact controllability of a single rigid body immersed in a 2D irrotational perfect incompressible fluid from an initial position and velocity to a final position and velocity was investigated.

The proof relies on a nonlinear method to solve linear perturbations of nonlinear equations associated with a quadratic operator having a non-zero non-degenerate critical point. Here this method is applied to a quadratic equation satisfied by a class of boundary controls, which is obtained by extending the reformulation of the Newton equations performed in the uncontrolled case in [2] to the case where a control acts on the external boundary. The quadratic operator mentioned above is then obtained as a part of the force terms on the moving rigid bodies due to the fluid motion driven by the control on the fixed external boundary. A class of controls for which this operator has a non-zero non-degenerate critical point is constructed by complex analysis methods.

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## Kinect-Based Gesture Recognition for micro:bit Robot Car Control

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Gesture recognition is a topic in computer science and language technology with the goal of interpreting human gestures via mathematical algorithms. In this paper we want to show, how to use gesture recognition to control a micro:bit based robot car. Using a Kinect sensor, the system detects the motion of the human user and creats the skeletal image of the body. Coordinate Geometry and different approximation methods are used to calculate the angles between the bones connecting the joints. In our project inverse kinematics makes use of the kinematics equations to determine the joint parameters that provide a desired position for each of the robot's end-effectors. The BBC micro:bit is an open source hardware ARM-based embedded system designed by the BBC for use in computer education. Building a micro:bit robot is an exciting way to learn how to code, and combined with Kinect, we can also provide easy control.

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## A hybrid CNN-SVM approach with dynamic time warping for time series classification

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Recently, convolutional neural networks (CNNs) have been successfully applied in many fields, including time series classification. However, in earlier years an important role played methods based on a dynamic time warping (DTW) distance. Therefore some researchers tried to combine the advantages of CNNs and DTW and create hybrid methods. In this work, we demonstrate a new approach which consists of three main stages. In the initial phase, a conventional CNN is trained. Then, based on weights from the first convolutional layer of the learned network, DTW distances are calculated. Finally, such extracted features are used for the training of a traditional classifier like Support Vector Machine (SVM). The use of SVM instead of another neural network should reduce the whole calculation time and ensure the more explainable model. We present initial results of this method for selected datasets.

## Acknowledgment

K.K. acknowledges funding from the European Union through the European Social Fund (grant POWR.03.02.00-00-I029).

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#### Boundary curves in the weak stability transition-region

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The weak capture represents the event where the Keplerian energy of the massless particle relative to one of the primaries changes its sign from positive to negative in the context of the restricted three-body problem. Belbruno (1987) introduced the notion of weak stability boundary (WSB) by designing transfer orbits from Earth to Moon. In this paper Belbruno also proposed an algorithmic definition of the WSB, where the initial conditions are classified to be stable or unstable according to weak stability criteria.

In the articles dedicated to the study of WSB, certain parameters (for example, the true anomaly of Earth, Moon or another planets, the initial eccentricity of the test particle, the initial direction of the test particle) are considered constant and the initial position  $\mathbf{r}_2$  of the test particle relative to the primary  $P_2$  is considered a variable parameter. The modulus of initial velocity  $\mathbf{v}_2(\mathbf{r}_2)$  of the massles particle relative to the primary  $P_2$  is determined, and then the weak stability of the orbit with the initial values  $(\mathbf{r}_2, \mathbf{v}_2(\mathbf{r}_2))$  is examined. In these cases, the independent variable is  $\mathbf{r}_2$  and the dependent variable is  $\mathbf{v}_2$ .

In this lecture, the velocity  $\mathbf{v}_2(\mathbf{r}_2)$  is also considered as an independent variable in the study of weak stability. Four boundary curves are defined in the weak stability transition-region (WSTR). As an application, the boundary curves of WSTR are numerically determined in the Sun-Earth system. The locations of the boundary curves are compared to the Earth-Moon mean distance.

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## Mean Field Games and Master Equations

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The theory of mean field games has been initiated around 15 years ago by Lasry-Lions on the one hand and by Huang-Malhamé-Caines on the other hand. The main goal of both groups (inspired by the mean field models from statistical physics) was to characterize limits of Nash equilibria of stochastic differential games, when the number of agents tends to infinity. Since then, this theory has witnessed a great success, both theoretically and from the point of view of applications.

In this talk we take a journey into this field, starting with the derivation of the main systems of PDEs, which characterize the mentioned limits of the equilibria. Then, we present the so-called master equation, which was first introduced by Lions. This is an infinite dimensional PDE set on the space of Borel probability measures, which encodes all the properties of the underlying game. Because of their infinite dimensional nature, many new challenges arise regarding the solvability of these equations. In the second half of the talk, we will discuss how different notions of convexity/monotonicity on the data could lead to the global in time well-posedness of these equations. Our main results in this direction have been obtained recently in collaboration with W. Gangbo (UCLA) on the one hand and with W. Gangbo, C. Mou (City U, Hong Kong) and J. Zhang (USC) on the other hand.

## A novel approach to Gröbner basis algorithms with implementations in GAP

#### Roland-Botond Miklosi, István Szöllősi

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The theory of the Gröbner bases has an important role in solving complex problems in mathematics and physics. The Gröbner basis is basically the generator system of a polynomial ideal with really good attributes and it is rather complex to calculate. Since the seminal work of Buchberger, much research has been conducted in order to develop an efficient algorithm to calculate the Gröbner basis. Most computer algebra systems use Faugère's F5 and F4 algorithms.

The GVW algorithm by Gao, Volny and Wang has a clearer theoretical description and is more efficient than the Faugère algorithms.

We created an own implementation for the GAP open source computer algebra system, based on the GVW algorithm, enhanced with optimizations and extensions. Our implementation is considerably faster than the built-in implementation currently used by GAP (which is based on Buchberger's algorithm).

The talk introduces briefly the theory of Gröbner bases and presents our own implementation based on the GVW algorithm, which we compare with other commonly known computer algebra systems.

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## On the Neumann fractional p-Laplacian

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We report on some recent results on the fractional p-Laplacian with (nonlocal) Neumann boundary conditons.

## Inspired by Fireworks: A Novel Swarm Intelligence Algorithm

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The explosion of fireworks inspired a new algorithm for global optimization of complex functions, the Fireworks Algorithm (FWA). The algorithm mimics the process of exploding fireworks. It is a swarm intelligence algorithm that explores a very large solution space by choosing a set of random points confined by some distance metric in the hopes that one or more of them will yield promising results (it explodes at one point and scatters more sparks around itself. Each spark location is evaluated, and the algorithm terminates if an optimal location was found otherwise it repeats with n new firework locations).

In this talk, I will briefly explain how the algorithm works, with various applications.

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## Comparing epidemiological models with the help of visualization dashboards

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In 2020, due to the COVID - 19 pandemic, various epidemiological models appeared in major studies [1, 2, 3, 4], which differ in terms of complexity, type, etc. In accordance with the hypothesis, a complex model, which takes into consideration more parameters, is more accurate and gives more reliable results than a simpler one.

In this paper we study three different epidemiological models: a SIR, a SEIR and a SEIR - type model. Our aim is to set up differential equation models, which rely on similar parameters, however, the systems of equation and number of parameters deviate from each other. A visualization dashboard<sup>1</sup> is implemented through this study, and thus, we are able not only to study the models but also to make users understand the differences between the complexity of epidemiological models, and ultimately, to share a more specific overview about these that are defined by differential equations [5].

In order to validate our results, we make a comparison between the three models and the empirical data from Eastern Italy and Wuhan, based on the infectious cases of COVID-19. To validate our results, we calculate the values of the parameters using the Least Square optimization algorithm.

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<sup>&</sup>lt;sup>1</sup>https://seir-visualisation.vercel.app/

## Algorithm visualization using human movement effect: It's your turn

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Computational thinking is one of the most essential skills in every person's life from the XXI. century [1]. Acquiring this is based on many important factors, such as the presentation of the educational material, learning methods and in what form (hearing, vision, human movement) or to what extent the user should be involved in the learning process.

As students are very interested in computer games these days and realistic illustrations are becoming more and more popular nowadays, presenting algorithms with the help of a virtual environment can provide an excellent opportunity for students to be part of a role-playing game as soon as they take the place of one of the dancers.

The aim of the present study is to present my previous research results in the field of computational thinking, interactivity and different algorithm types. Considering this previous work, we have used a novel online learning tool (AlgoRythmics) which includes visualizations of ten basic computer algorithms (searching and sorting strategies) and three interactivity levels: no-interactivity (users are only independent observers), half-interactivity (at specific key moments students need to specify the next correct movement) and fullinteractivity (students need to control the animation process). It is also known that the efficiency of this can be influenced by many factors such as the type of the algorithms. We decided to focus on the AlgoRythmics illustration of three sorting algorithms (shell, selection and quick) and, more specifically, on the influence that the degree of interactivity has on students' learning.

Based on these aspects and experiences my aim is to establish and create the structure of a virtual environment in which learners are no longer just external observers of the steps of algorithms, but can space and control the algorithm visualization themselves.

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## Large semigames

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Large games are such games where both the player set and the players' strategy sets can be infinite. We consider various mixed extensions of large games, the properties of the mixed extensions and the relations among them. We introduce the notion of semigames where the payoffs are not necessarily defined.

We also generalize the notion of Nash equilibrium for semigames and show that every semigame has a generalized Nash equilibrium.

## On certain local and nonlocal (p,q) systems in $\mathbb{R}^N$ with critical and Hardy terms

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Motivated by important applications in nonlinear elasticity, recently great attention has been devoted to the study of local and nonlocal nonlinear problems with (p,q) growth conditions.

We present existence results for a class of parametric (p, q) systems with critical and Hardy terms in  $\mathbb{R}^N$ , provided that the parameter is sufficiently large. The interest is twofold: on one hand, the simultaneous presence of critical terms, Hardy terms and the fact that the systems are studied in the whole  $\mathbb{R}^N$  cause, roughly speaking, a *triple loss of compactness* which dramatically affects the applicability of standard variational methods. On the other hand, since we treat both the local and the nonlocal version of the system, the comparison of the results obtained for fractional Laplacian operators with their local counterpart is noteworthy.

The results of the talk are obtained jointly with Letizia Temperini.

#### Strongly CS-Rickart objects in abelian categories

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Motivated by the work of Abyzov and Nhan on (dual) CS-Rickart modules [1], we introduced in [3] (dual) CS-Rickart objects in abelian categories as a common generalization of (dual) Rickart objects and extending (lifting) objects [2]. The concepts of (dual) Rickart objects and extending (lifting) objects in abelian categories may be specialized to those of (dual) strongly Rickart objects and strongly extending (lifting) objects by restricting the definition from direct summands to fully invariant direct summands. We present some general properties and examples, with emphasis on direct summands, (co)products of (dual) strongly relative CS-Rickart objects and classes all of whose objects are (dual) strongly self-CS-Rickart.

This is joint work with Septimiu Crivei (Babeş-Bolyai University).

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## Correlated equilibrium notions of interval valued bimatrix games

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In this paper, we introduce the notion of correlated and strong correlated equilibrium in bimatrix games, where the payoffs of the players are given by intervals.

The informal definition of correlated equilibrium is: each player chooses his/her action according to his/her observation of the same public signal. A strategy assigns an action to every possible observation a player can make. If no player would want to deviate from the recommended strategy (assuming the others don't deviate), the distribution is called a correlated equilibrium.

We give a characterization of correlated and strong correlated equilibriums. The correlated equilibrium (strong correlated equilibrium) is a generalization of Nash equilibrium (strong Nash equilibrium).

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#### Almost everywhere convergence of sequences of $(C,\alpha)$ means of *m*-adic Fourier series of integrable functions

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This talk investigates convergence questions on compact totally disconnected groups of the *m*-adic integers. It is known that the Fejér means – with respect to the product system of normed coordinate functions of continuous irreducible unitary representations of the coordinate groups – of an integrable function on these groups converge to the function a.e. In this talk we look at the above convergence for  $(C,\alpha)$  means. This is a joint work with Gy. Gát.

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## Torsional creep problems involving Grushin-type operators

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The asymptotic behaviour of solutions for a family of torsional creep problems involving the Grushin *p*-Laplacian is investigated. In particular, our results complement some earlier works on the topic by L. E. Payne & G. A. Philippin [3], B. Kawohl [2] and T. Bhattacharya, E. DiBenedetto and J. Manfredi [1]. This is based on a joint work with Mihai Mihailescu. This presentation is partially supported by CNCS-UEFISCDI Grant No. PN-III-P1-1.1-TE-2019-0456.

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#### On some particular tame Hall polynomials

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Classical Hall algebras associated with discrete valuation rings were introduced by Steinitz and Hall to provide an algebraic approach to the classical combinatorics of partitions. The multiplication is given by Hall polynomials which play an important role in the representation theory of the symmetric groups and the general linear groups. In 1990 Ringel defined Hall algebras for a large class of rings, namely finitary rings, including in particular path algebras of quivers over finite fields. Far reaching analogues of the classical ones, these Ringel-Hall algebras provided a new approach to the study of quantum groups using the representation theory of finite dimensional algebras. They can also be used successfully in the theory of cluster algebras or to investigate the structure of the module category.

In case of Ringel-Hall algebras corresponding to Dynkin quivers and tame quivers we know due to Ringel and Hubery, that the structure constants of the multiplication are again polynomials in the number of elements of the base field. These are the generalized Hall polynomials. If we are looking at Hall polynomials associated to indecomposable modules, the classical ones are just 0 or 1, the generalized ones in the Dynkin case are also known and have degree up to 5, however we do not have too much information about the generalized ones in the tame case.

Let k be an arbitrary field and Q a tame quiver of type  $\tilde{D}_4$ . Consider the path algebra kQ and the category of finite dimensional right modules mod-kQ.

We describe tools and techniques used to determine all the Hall polynomials  $F_{xy}^z$  associated to indecomposable modules  $x, y, z \in \text{mod-}kQ$ . As a result we obtain a first list of tame Hall polynomials involving indecomposables of absolute defect different from 0 or 1.

## Galois Connections in Subgroup Lattices

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Given two partially ordered sets  $(P, \leq)$  and  $(Q, \sqsubseteq)$ , a pair of mappings  $f : P \to Q$ and  $g : Q \to P$  is called a *Galois connection* (or Galois correspondence) if the mappings satisfy the two conditions  $x \leq (g \circ f)x$  and  $(f \circ g)y \sqsubseteq y$  for any pair of elements x, yin P and Q, respectively. An equivalent characterization states that the inverse image under f of every principal down-set  $\{y' \in Q | y' \sqsubseteq y\}$  of Q is a principal down-set of P, and this correspondence determines g uniquely. Therefore, the classification of Galois connections is closely related to the structure of the underlying lattices. We review some known facts about subgroup lattices L(G) of p-primary groups G from the perspective of Galois connections  $f, g : L(G) \to L(G)$  and point to interesting related problems.

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