Mean Field Games and Master Equations

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The theory of mean field games has been initiated around 15 years ago by Lasry-Lions on the one hand and by Huang-Malhamé-Caines on the other hand. The main goal of both groups (inspired by the mean field models from statistical physics) was to characterize limits of Nash equilibria of stochastic differential games, when the number of agents tends to infinity. Since then, this theory has witnessed a great success, both theoretically and from the point of view of applications.

In this talk we take a journey into this field, starting with the derivation of the main systems of PDEs, which characterize the mentioned limits of the equilibria. Then, we present the so-called master equation, which was first introduced by Lions. This is an infinite dimensional PDE set on the space of Borel probability measures, which encodes all the properties of the underlying game. Because of their infinite dimensional nature, many new challenges arise regarding the solvability of these equations. In the second half of the talk, we will discuss how different notions of convexity/monotonicity on the data could lead to the global in time well-posedness of these equations. Our main results in this direction have been obtained recently in collaboration with W. Gangbo (UCLA) on the one hand and with W. Gangbo, C. Mou (City U, Hong Kong) and J. Zhang (USC) on the other hand.