First and second order Caffarelli-Kohn-Nirenberg type inequalities: sharp constants and minimizers.

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We discuss sharp first and second order inequalities of Caffarelli-Kohn-Nirenberg (CKN) type in the euclidian space $\mathbb{R}^N$, where N denotes the dimension.

Firstly, we provide very short and self-contained proofs of a refined version of the main results by F. Catrina and D. Costa (J. Differential Equations 2009). Our results are sharp and minimizers are obtained in suitable functional spaces.

Secondly, we analyze second order CKN inequalities. This is equivalent to the study of uncertainty principles for special classes of vector fields. In particular, we show that when switching from scalar fields $u : \mathbb{R}^N \to \mathbb{C}$ to vector fields of the form $\vec{u} := \nabla U$ (U being a scalar field) the best constant in the Heisenberg Uncertainty Principle (HUP) increases from $\frac{N^2}{4}$ to $\frac{(N+2)^2}{4}$, and the optimal constant in the Hydrogen Uncertainty Principle (HyUP) improves from $\frac{(N-1)^2}{4}$ to $\frac{(N+1)^2}{4}$. We also provide minimizers for the improved sharp constants. As a consequence of our results we answer to an open question of Maz'ya (Integral Equations Operator Theory 2018, Section 3.9) in the case $N = 2$ regarding the HUP for divergence free vector fields.

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