Computer assisted proof of the tree module property for representations of Euclidean quivers

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We present a computational method used for the classification of indecomposable objects in the category of representations of Euclidean quivers. The method may be used for explicitly describing and proving the so-called tree module property (or tree representations). Recall that tree representations are indecomposable and can be exhibited using matrices involving only the elements 0 and 1, with the total number of ones being exactly $d-1$ (where d is the length of the module). Due to a result of Ringel (see [3]) the existence of tree representations is guaranteed when the module is exceptional (indecomposable and without self-extensions).

Using the novel method, we were able to give a complete and general list of tree representations corresponding to exceptional modules over the path algebra of the canonically oriented Euclidean quiver $\tilde{\mathbb{E}}_6$. Moreover, all these representations remain valid over any base field, this fact answering an open question raised/suggested in [3]. The proof (involving induction and symbolic computation with block matrices) was partially generated by a purposefully developed computer software. We give some details on the inner workings of this software developed in Clean (a general-purpose purely functional computer programming language).

References

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