Brezis-Poincaré-Vázquez inequalities on non-euclidean structures

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In this talk we investigate an interpolation inequality between the Brezis-Vázquez and Poincaré inequalities (shortly, BPV inequality) on nonnegatively curved spaces. We first prove that the BPV inequality holds on any Minkowski space, by fully characterizing the existence and shape of its extremals. We then prove that if a complete Finsler manifold with nonnegative Ricci curvature supports the BPV inequality, then its flag curvature is identically zero. In particular, we deduce that a Berwald space of nonnegative Ricci curvature supports the BPV inequality if and only if it is isometric to a Minkowski space. Our arguments explore fine properties of special functions, comparison principles, and anisotropic symmetrization on Minkowski spaces. As an application, we characterize the existence of nonzero solutions for a quasilinear PDE involving the Finsler-Laplace operator and a Hardy-type singularity on Minkowski spaces where the sharp BPV inequality plays a crucial role. Talk based on the paper [1].

References

[1] A. Kristály, A. Szakál, Interpolation between Brezis-Vzquez and Poincar inequalities on non-negatively curved spaces: sharpness and rigidities. *J. Differential Equations* 266 (2019), no. 10, 6621–6646.