Extensions of Linear Cellular Automata

Adriana Popovici

Department of Computer Science, West University of Timişoara

adriana.popovici@e-uvt.ro

In the recent years it was often more and more difficult to describe complex real life systems using only discrete state space cellular automata (in short, CAs). Many applications based on CAs with continuous state space were considered. In this context we proposed a general CA model in which the state space is a Hilbert space and the local transition function is a bounded linear map. More precisely, a *linear cellular automaton* (LCA) is a triple $\mathcal{A} = (\mathcal{H}, N, \delta)$ where $\mathcal{H}$ (the *state space*) is a complex Hilbert space (finite or infinite dimensional), $N = N_1 = \{-1, 0, 1\}$ (the *neighborhood*) and $\delta : \mathcal{H}^3 \to \mathcal{H}$ (the *local rule*) is a linear and bounded map. The *configuration space* is, in this case, the Hilbert space $\mathcal{C}_{\mathcal{A}} := \ell^2_{\mathbb{Z}}(\mathcal{H})$ of all square summable sequences $(h_n)_{n\in\mathbb{Z}}$ of vectors in $\mathcal{H}$. The *global transition function*, which describes the CA evolution, is defined as

$$\mathcal{C}_{\mathcal{A}} \ni c = (h_n)_{n\in\mathbb{Z}} \mapsto G_{\mathcal{A}}(c) := (\delta(h_{n+N}))_{n\in\mathbb{Z}} \in \mathcal{C}_{\mathcal{A}}.$$

In one of the first studies on quantum computing R.P. Feynman suggested the necessity in quantizing the CA model. There were two main reasons for introducing quantum cellular automata (QCAs): firstly, they would eliminate the need for an external control (which is an important source of decoherence) and, secondly, they would provide useful models for quantum computation. A one-dimensional QCA can be naturally defined as a unitary transformation which acts over a line of finite dimensional quantum systems, it is translation-invariant and causal. In our approach QCAs are introduced and studied in an infinite dimensional framework. QCAs were used in the recent years as quantum mechanical models of computing, strengthening the bridge between computer science and theoretical physics. We want to mention that QCAs are reversible CAs because their evolution is determined by a unitary transformation. In this context it is important to find larger classes of cellular automata with a behavior at least comparable, as concerning the application point of view, with the behavior of reversible CAs. The class of CAs which are dilatable (or extensible) to a reversible CA represent a good example in this direction. We can perform computations and recover the past information (when needed) using only the dilation CA and, finally, compress the results into the configuration space of the original CA.

We chose to use continuous state CAs because, in real life systems, one can rarely find situations described by only a finite set of states. In order to obtain better results provided by some powerful tools of functional analysis we decided to add additional structure on the state space (a Hilbert space) and on the local rule (a linear and bounded map). We introduce a notion of dilatability between two LCAs and relate it with the notion of (power) dilatability between the corresponding global transition functions. If the local rule of a given LCA $\mathcal{A}$ is a row contraction we show that $\mathcal{A}$ can be dilated to a LCA having a local rule with isometric components. We prove that a partial isometric LCA can be dilated to a quantum LCA which is reversible. In particular, any isometric LCA $\mathcal{A}$ can be dilated to a quantum LCA $\mathcal{B}$ such that the global rule of $\mathcal{B}$ extends the global rule of $\mathcal{A}$.

The talk is based on a joint work with Dan Popovici.