Decompositions for triples of commuting isometries

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We discuss various types of decompositions for triples $V = (V_1, V_2, V_3)$ of commuting isometries. Our first results involve pairs of the form (V_iV_j, V_k) for $i, j, k \in \{1, 2, 3\}$ with $i \neq j \neq k \neq i$. Assuming that the bi-isometries (V_1V_2, V_3) and (V_2V_3, V_1) have Wold-Słociński decompositions (in short, WSDs) [3] we prove that there exists an orthogonal decomposition of the form

$$\mathcal{H} = \mathcal{H}_{uuu} \oplus \mathcal{H}_{uus} \oplus \mathcal{H}_{suu} \oplus \mathcal{H}_{usu} \oplus \mathcal{H}_{ssu} \oplus \mathcal{H}_{uss} \oplus \mathcal{H}_{s\cdot s}$$

into subspaces $\mathcal{H}_{\alpha_1\alpha_2\alpha_3}$ ($\alpha_1, \alpha_3 \in \{u, s\}$, $\alpha_2 \in \{u, s, \cdot\}$) which reduce V_i to a unitary operator if $\alpha_i = u$, respectively to a unilateral shift if $\alpha_i = s$, $i \in \{1, 2, 3\}$. The converse is also true. Our next aim is to provide necessary and sufficient conditions ensuring an orthogonal decomposition of the form $\mathcal{H}_{s\cdot s} = \mathcal{H}_{sus} \oplus \mathcal{H}_{sss}$ into reducing subspaces for V such that $V_2|_{\mathcal{H}_{sus}}$ is a unitary operator and $V_2|_{\mathcal{H}_{sss}}$ is a unilateral shift. In other words, under such assumptions, the decomposition above can be completed to a WSD for the triple $V = (V_1, V_2, V_3)$. More precisely, the triple V has a WSD if and only if the pairs (V_1V_2, V_3) and (V_2V_3, V_1) have WSDs and the subspace $\mathcal{H}_s^{V_1} \cap \mathcal{H}_s^{V_2} \cap \mathcal{H}_s^{V_3}$, respectively $\mathcal{H}_s^{V_2}$, is invariant under the isometries V_1 and V_3 (for a given isometric operator W on $\mathcal{H}$, $\mathcal{H}_s^W$ denotes the corresponding shift part in the Wold-Halmos decomposition of W [1]). Equivalently, the pairs (V_1V_2, V_3), (V_2V_3, V_1) and (V_1V_3, V_2) have WSDs. Certain examples are provided for illustrative purposes.

The final part contains several applications of the case $n = 3$ with the aim of generating Wold-type decomposions for arbitrary commuting systems of isometries.

The talk is based on a joint work with T. Bînzar, Z. Burdak, C. Lăzureanu şi M. Słociński [2].

References

[1] P.R. Halmos, Shifts on Hilbert spaces, *J. Reine Angew. Math.* **208** (1961) 102–112.

[2] T. Bînzar, Z. Burdak, C. Lăzureanu, D. Popovici, M. Słociński, WoldSłociński decompositions for commuting isometric triples, *J. Math. Analysis and Appl.* **472** 2 (2019) 1660–1677.

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