Theoretical and simulation results on a network evolution model driven by a branching process

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We consider a network evolution model driven by a continuous time branching process. Our model is an extension of the model studied by T. F. Móri and S. Rokob [1]. Our initial network is an N-clique, where $N \geq 2$ is a fixed integer (a complete graph with N vertices is called an N-clique). The evolution of our network is the following. For any fixed N-clique the reproduction time is given by a Poisson process. If in the Poisson process a new individual is born, then in our model the N-clique produces a new vertex. This new vertex will be attached to some vertices of its mother N-clique. The number of these attaching edges is random and the vertices to be attached to the new vertex are chosen uniformly at random. Therefore at any reproduction time might be one new N-clique or N new N-cliques are created (it is possible that there is no new N-clique). The mother N-clique continues this reproduction process and her new-born children N-cliques also start reproduction by the same way. The lifetime of an individual depends on its biological age that is the number of its children. After an N-clique stops reproduction, there are two ways to handle this. Either the N-clique remains an inactive part of the network or it can be considered dead and it is removed from the network. We obtain results on the number of N-cliques and also the number of vertices and edges in the network. We present both mathematical theorems and simulation results. We show numerical results for $N = 2, 3$.

References

[1] T. F. Móri and S. Rokob: *A random graph model driven by time-dependent branching dynamics.* Annales Univ. Sci. Budapest. **46** (2017) 191–213.