The Monotonicity of the Principal Eigenvalue of the p-Laplace Operator

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In this talk we present some results obtained in collaboration with Marian Bocea and Julio D. Rossi.

First, we deal with monotonicity with respect to p of the first eigenvalue of the p-Laplace operator on Ω subject to the homogeneous Dirichlet boundary condition. For any fixed integer $D > 1$ we show that there exists $M \in [e^{-1}, 1]$ such that for any open, bounded, convex domain $\Omega \subset \mathbb{R}^D$ with smooth boundary for which the maximum of the distance function to the boundary of Ω is less than or equal to M, the first eigenvalue of the p-Laplace operator on Ω subject to the homogeneous Dirichlet boundary condition is an increasing function of p on $(1, \infty)$. Moreover, for any real number $s > M$ there exists an open, bounded, convex domain $\Omega \subset \mathbb{R}^D$ with smooth boundary which has the maximum of the distance function to the boundary of Ω equal to s such that the principal eigenvalue of the p-Laplacian is not a monotone function of $p \in (1, \infty)$.

Second, we deal with monotonicity with respect to p of the first positive eigenvalue of the p-Laplace operator on Ω subject to the homogeneous Neumann boundary condition. For any fixed integer $D > 1$ we show that there exists $N \in [2e^{-1}, 2]$ such that for any open, bounded, convex domain $\Omega \subset \mathbb{R}^D$ with smooth boundary for which the diameter of Ω is less than or equal to N, the first positive eigenvalue of the p-Laplace operator on Ω subject to the homogeneous Neumann boundary condition is an increasing function of p on $(1, \infty)$. Moreover, for each real number $s > N$ there exists a sequence of open, bounded, convex domains $\{\Omega_n\}_n \subset \mathbb{R}^D$ with smooth boundaries for which the sequence of the diameters of Ω_n converges to s, as $n \to \infty$, and for each n large enough the first positive eigenvalue of the p-Laplace operator on Ω_n subject to the homogeneous Neumann boundary condition is not a monotone function of p on $(1, \infty)$.

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