Character triples and group graded equivalences

Minuță Virgilius-Aurelian

Babeș-Bolyai University of Cluj-Napoca, Department of Mathematics

minuta.aurelian@math.ubbcluj.ro

Let N be a normal subgroup of G, G' a subgroup of G, and N' a normal subgroup of G'. We assume that $N' = G' \cap N$ and $G = G'N$, hence $\bar{G} := G/N \simeq G'/N'$. Let $b \in Z(\mathcal{O}N)$ and $b' \in Z(\mathcal{O}N')$ be $\bar{G}$-invariant block idempotents. We denote $A := b\mathcal{O}G$ and $A' := b'\mathcal{O}G'$. Then A and A' are strongly $\bar{G}$-graded algebras, with 1-components B and B' respectively. Additionally, assume that $C_G(N) \subseteq G'$, and denote $C := \mathcal{O}C_G(N)$, which is regarded as a $\bar{G}$-graded $\bar{G}$-acted algebra.

In [2, Definition 2.7.], Britta Späth considers a relation $\geq_c$ between the character triples (G, N, θ) and (G', N', θ'), where θ is $\bar{G}$-invariant irreducible character belonging to the block b and θ' is a $\bar{G}$-invariant irreducible character belonging to the block b'.

We introduce $\bar{G}$-graded (A, A')-bimodules over C and we study Morita equivalences between A and A' induced by such bimodules.

We prove that if θ corresponds to θ' under a $\bar{G}$-graded Morita equivalence over C, then $(G, N, \theta) \geq_c (G', N', \theta')$.

We also show that an analogue of the so-called "butterfly theorem" [2, Theorem 2.16] holds for $\bar{G}$-graded Morita equivalences over C.

References

[1] A. Marcus, V.A. Minuță, *Group graded endomorphism algebras and Morita equivalences*, preprint 2019, 1-8;

[2] B. Späth, *Reduction theorems for some global-local conjectures*, In: Local Representations Theory and Simple Groups, EMS Ser. Congr. Rep., Eur. Math. Soc., Zürich (2018), 23-62.