Rayleigh-Taylor turbulence via convex integration

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We study the mixing of two different density perfect incompressible fluids subject to gravity, when the heavier fluid is on top. It is well-known in the physics literature that in such a case an instability forms on the interface between the fluids which eventually evolves into turbulent mixing.

The mathematical model is given by the inhomogeneous incompressible Euler equation:

$$\begin{aligned} \partial_t(\rho v) + \mathrm{div}\ (\rho v \otimes v) + \nabla p = -\rho g e_2, \\ \mathrm{div}\ v = 0, \\ \partial_t \rho + \mathrm{div}\ (\rho v) = 0, \end{aligned}$$

where $\rho : \mathbb{R}^2 \times [0,T] \to \mathbb{R}$ denotes the fluid density, $v : \mathbb{R}^2 \times [0,T] \to \mathbb{R}^2$ is the velocity field, respectively $p : \mathbb{R}^2 \times [0,T] \to \mathbb{R}$ is the pressure field, $g > 0$ is the gravitational constant and $e_2 = (0,1)$. We consider initial data $v_0 \equiv 0$ and

$$\rho_0(x) = \begin{cases} \rho_+ \text{ when } x_2 > 0, \\ \rho_- \text{ when } x_2 \leq 0, \end{cases},$$

with $\rho_+ > \rho_- > 0$.

In the spirit of the celebrated results by De Lellis and Székelyhidi Jr. [2] for the homogeneous incompressible Euler equation, we develop a convex integration strategy for this model to obtain non-uniqueness of weak solutions. We also prove that in a particular class of weak solutions, the maximal diameter of the mixing zone (up to a constant) grows like $\mathcal{A} g t^2$ (where $\mathcal{A} = \frac{\rho_+ - \rho_-}{\rho_+ + \rho_-}$ denotes the Atwood number), a property that has also been observed by the physics community [1].

References

[1] G. Boffetta, A. Mazzino, Incompressible Rayleigh–Taylor Turbulence, Annual Review of Fluid Mechanics, Vol. 49 (January 2017), 119–143.

[2] C. De Lellis, L. Székelyhidi Jr., The Euler equations as a differential inclusion. Ann. Math. (2) 170, 3 (2009), 1417–1436.