A characterization related to Schrödinger equations

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We consider the following Schrödinger equation

$$\begin{cases} -\Delta u + V(x)u = \lambda\alpha(x)f(u), & \text{in } \mathbb{R}^N \\ u > 0, & \text{in } \mathbb{R}^N \\ u \to 0, & \text{as } |x| \to \infty \end{cases} \qquad (\mathcal{P}_\lambda)$$

where $N > 2$, $\alpha : \mathbb{R}^N \to]0, +\infty[$ is a function in $L^\infty(\mathbb{R}^N) \cap L^1(\mathbb{R}^N)$, $f : [0, +\infty[\to [0, +\infty[$ is a continuous function with $f(0) = 0$ and subcritical growth, $V : \mathbb{R}^N \to]0, +\infty[$ is a continuous and coercive potential.

Motivated by the characterization proved by Ricceri in [2] for a two point boundary value problem, we prove

Theorem 1 *Assume that for some $a > 0$ the function $\xi \to \frac{F(\xi)}{\xi^2}$ is non-increasing in $]0, a]$. Then, the following conditions are equivalent:*

(i) for each $b > 0$, the function $\xi \to \frac{F(\xi)}{\xi^2}$ is not constant in $]0, b]$;

(ii) for each $r > 0$, there exists an open interval $I \subseteq]0, +\infty[$ such that for every $\lambda \in I$, problem $(\mathcal{P}_\lambda)$ has a solution $u_\lambda \in H^1(\mathbb{R}^N)$ satisfying $\int_{\mathbb{R}^N} \left(|\nabla u_\lambda|^2 + V(x)u_\lambda^2\right) dx < r$.

Based on the joint work [1].

References

[1] F. Faraci, Cs. Farkas, A characterization related to Schrödinger equations on Riemannian manifolds, submitted.

[2] B. Ricceri, A characterization related to a two-point boundary value problem, *J. Nonlinear Convex Anal.*, **16** (2015) 79–82.