A characterization related to Schrödinger equations

Francesca Faraci
Department of Mathematics and Computer Sciences
University of Catania, Italy
ffaraci@dmi.unict.it

We consider the following Schrödinger equation

\[
\begin{cases}
-\Delta u + V(x)u = \lambda \alpha(x)f(u), & \text{in } \mathbb{R}^N \\
u > 0, & \text{in } \mathbb{R}^N \\
u \to 0, & \text{as } |x| \to \infty
\end{cases}
(\mathcal{P}_\lambda)
\]

where $N > 2$, $\alpha : \mathbb{R}^N \to [0, +\infty]$ is a function in $L^\infty(\mathbb{R}^N) \cap L^1(\mathbb{R}^N)$, $f : [0, +\infty[ \to [0, +\infty]$ is a continuous function with $f(0) = 0$ and subcritical growth, $V : \mathbb{R}^N \to ]0, +\infty[$ is a continuous and coercive potential.

Motivated by the characterization proved by Ricceri in [2] for a two point boundary value problem, we prove

**Theorem 1** Assume that for some $a > 0$ the function $\xi \to \frac{F(\xi)}{\xi^2}$ is non-increasing in $[0, a]$. Then, the following conditions are equivalent:

(i) for each $b > 0$, the function $\xi \to \frac{F(\xi)}{\xi^2}$ is not constant in $[0, b]$;

(ii) for each $r > 0$, there exists an open interval $I \subseteq [0, +\infty[$ such that for every $\lambda \in I$, problem $(\mathcal{P}_\lambda)$ has a solution $u_\lambda \in H^1(\mathbb{R}^N)$ satisfying $\int_{\mathbb{R}^N} \left( |\nabla u_\lambda|^2 + V(x)u_\lambda^2 \right) dx < r$.

Based on the joint work [1].

**References**
