Intrinsic geometric inequalities on sub-Riemannian structures

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We establish a weighted pointwise Jacobian determinant inequality on corank 1 Carnot groups (e.g. Heisenberg groups) related to optimal mass transportation akin to the work of Cordero-Erausquin, McCann and Schmuckenschläger [4]. The weights appearing in our expression are distortion coefficients that reflect the delicate sub-Riemannian structure of our space including the presence of abnormal geodesics. Our inequality interpolates in some sense between Euclidean and sub-Riemannian structures, corresponding to the mass transportation along abnormal and strictly normal geodesics, respectively. As applications, entropy, Brunn-Minkowski and Borell-Brascamp-Lieb inequalities are established. These results refute a general point of view, according to which no geometric inequalities can be derived by optimal mass transportation on singular spaces. In particular, our results complement the theory of Lott and Villani [5] and Sturm [6, 7], where the Riemannian manifolds constituted the modeling structures. Furthermore, our approach is the first step to perform the "grande unification" of the three geometries: Riemannian, Finslerian and the sub-Riemannian, suggested by Villani [8]. The talk is based on three joint papers with Z. Balogh (Bern) and K. Sipos (Bern), see [1]-[3].

References

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