The proof of the Brannan conjecture in particular cases

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Abstract

We will prove the Brannan conjecture for particular values of the parameter. The basic tool of the study is an integral representation published in a recent work.

We consider the following Mac-Laurin development

$$\frac{(1+xz)^{\alpha}}{(1-z)^{\beta}} = \sum_{n=0}^{\infty} A_n(\alpha, \beta, x) z^n \qquad (1)$$

where $\alpha > 0,\ \beta > 0,\ x = e^{i\theta},\ \theta \in [-\pi, \pi]$, and $z \in U$. It is easily seen, that the radius of convergence of the series (1) is equal to 1. In [5] the author conjectured, that if $\alpha > 0,\ \beta > 0$ and $|x| = 1$, then

$$|A_{2n-1}(\alpha, \beta, x)| \leq A_{2n-1}(\alpha, \beta, 1),$$

where n is a natural number. Partial results regarding this question have been proved in [1], [2], [5], [8].
The case $\beta = 1$, $\alpha \in (0, 1)$ is stil open. Regarding this case have been obtained partial results in [3], [4], [6], [7]. We are going to prove some partial results regarding the case $\beta = 1$, and $\alpha \in (0, 1)$. We will use an integral representation which have been proved in [3], and we will prove the conjecture in case $|\arg(x)| \leq \frac{2\pi}{3}$, $\beta = 1$, and $\alpha \in (0, 1)$.

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