An overview of the applications of certain minimax theorems

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In this lecture, I will highlight the great flexibility of certain minimax theorems which allows one to get a series of consequences in different fields. Here are two samples:

Theorem 1 *Every non-empty uniquely remotal compact subset of any normed space is a singleton.*

Theorem 2 *Let $a \geq 0$, $b > 0$, let $\Omega \subset \mathbf{R}^n$ be a smooth bounded domain, with $n \geq 4$, and let $p \in \left]0, \frac{n+2}{n-2}\right[$.*
Then, for each $\lambda > 0$ large enough and for each convex set $C \subseteq L^2(\Omega)$ whose closure in $L^2(\Omega)$ contains $H^1_0(\Omega)$, there exists $v^* \in C$ such that the problem

$$\begin{cases} -\left(a + b\int_\Omega |\nabla u(x)|^2 dx\right)\Delta u = |u|^{p-1}u + \lambda(u - v^*(x)), & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega \end{cases}$$

has at least three solutions, two of which are global minima in $H^1_0(\Omega)$ of the functional

$$u \to \frac{a}{2}\int_\Omega |\nabla u(x)|^2 dx + \frac{b}{4}\left(\int_\Omega |\nabla u(x)|^2 dx\right)^2 - \frac{1}{p+1}\int_\Omega |u(x)|^{p+1} dx - \frac{\lambda}{2}\int_\Omega |u(x) - v^*(x)|^2 dx.$$

A very challenging problem is as follows: does Theorem 2 hold for $n > 4$ and $p = \frac{n+2}{n-2}$?