An overview of the applications of certain minimax theorems

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In this lecture, I will highlight the great flexibility of certain minimax theorems which allows one to get a series of consequences in different fields. Here are two samples:

**Theorem 1** Every non-empty uniquely remotal compact subset of any normed space is a singleton.

**Theorem 2** Let \( a \geq 0, b > 0 \), let \( \Omega \subset \mathbb{R}^n \) be a smooth bounded domain, with \( n \geq 4 \), and let \( p \in \left[ 0, \frac{n+2}{n-2} \right] \). Then, for each \( \lambda > 0 \) large enough and for each convex set \( C \subseteq L^2(\Omega) \) whose closure in \( L^2(\Omega) \) contains \( H^1_0(\Omega) \), there exists \( v^* \in C \) such that the problem

\[
\begin{cases}
- \left( a + b \int_{\Omega} |\nabla u(x)|^2 dx \right) \Delta u = |u|^{p-1}u + \lambda (u - v^*(x)), & \text{in } \Omega \\
\quad u = 0, & \text{on } \partial \Omega
\end{cases}
\]

has at least three solutions, two of which are global minima in \( H^1_0(\Omega) \) of the functional

\[
u \to \frac{a}{2} \int_{\Omega} |\nabla u(x)|^2 dx + \frac{b}{4} \left( \int_{\Omega} |\nabla u(x)|^2 dx \right)^2 - \frac{1}{p+1} \int_{\Omega} |u(x)|^{p+1} dx - \frac{\lambda}{2} \int_{\Omega} |u(x) - v^*(x)|^2 dx.
\]

A very challenging problem is as follows: does Theorem 2 hold for \( n > 4 \) and \( p = \frac{n+2}{n-2} \)?