A Generalized Interior Approach to Constrained Set-Valued Duality

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This presentation addresses a new approach to duality for constrained set-valued optimization problems, by means of solutions defined with the help of the nonempty quasi interior of a convex cone, employing a set-type criterion. Weak duality theorems and theorems containing optimality conditions are presented, along with an application.

Form a theoretical point of view, a general constrained set-valued optimization problem

$$\min_{G(x)\cap -C\neq\emptyset} F(x), \qquad (P_0^{sv})$$

can be approached either by using the so-called **vector criterion**, or by developing the so-called **set criterion**.

Several authors tackled the set-valued optimization theory from perspectives involving the extension of results concerning vector-valued functions. The vector criterion employs the determination of vector-like efficient points ((Pareto)-efficient, weakly-efficient, strongly-efficient, etc.) within the entire set

$$\mathcal{F} = \bigcup \{F(x) : x \in X, G(x) \cap -C \neq \emptyset\}.$$

This means that $\overline{x} \in X$ such that $G(\overline{x}) \cap -C \neq \emptyset$, is an efficient solution to (P_0^{sv}) if there exists an $\overline{y} \in F(\overline{x})$ such that $\overline{y}$ is an efficient element to the set $\mathcal{F}$. Such an approach can be found in H. W. Corley [3], T. Tanino [12], T. Tanino and Y. Sawaragi [13], W. Song [10], [11]. In Chapter 7 of R. I. Boţ, S. M. Grad and G. Wanka [2] is presented a survey on set-valued duality using different vector-type extended notions.

The **set criterion** involves a direct comparison of the sets $F(x)$, for all $x \in X$, $G(x) \cap -C \neq \emptyset$ and it is based on an ordering relation on $\mathcal{P}(Y)$ rather than on an order on Y, case encountered when working with a vector criterion. This is the reason why, at times, the set criterion is also called the **natural criterion** in set-valued optimization. Connected to it, T. Kuroiwa published a series of articles [6], [7], [8]. We also mention Kuroiwa D., Tanaka T. and Truong X. D. H. [9]. Kuroiwa's approach was expanded in several recent papers written by E. Hernández and L. Rodríguez-Marin [5] and M. Alonso and L. Rodríguez-Marin [1].

We propose a new duality perspective for constained set-valued optimization problems, by employing efficient solutions defined with the help of the quasi interior of a convex cone, continuing the theory developed in [4] for unconstrained set-valued optimization problems.

First familiarizes the audience with the notions and results taken from the specialized literature. We present definitions and characterization properties for the quasi relative interior and quasi interior of a set. Two set-relations defined with the help of the quasi interior are presented along with some of their properties. A qi-conjugate function associated with a set-valued function, and a qi-subdifferential, in analogy to the conjugate functions and subdifferential from scalar optimization are mentioned as well.

Using a general perturbation approach, we construct a new set-valued duality theory for constrained problems, employing qi-efficient solutions. Our results are more general than those of E. Hernández and L. Rodríguez-Marin [5] for weak efficieny, since the quasi interior of a set is a more general notion than the interior.

An application of our set-valued Lagrange duality theorem by means of qi-efficiency, application stated in $\ell^2(\mathbb{R})$, is presented at last.

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