Algebraic characterization of temporal logics on forests

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A *forest* is an ordered finite vector of finite unranked, ordered *trees* over some finite *alphabet A*, i.e. when $t_1, \ldots, t_n$ are trees, then $\sum_{i=1}^{n} t_i$ is a forest and when s is a forest and $a \in A$ is a letter, then $a(s)$ is a tree.

A finite *forest automaton* is a device $\mathbb{M} = (Q, +, 0, A)$ where $(Q, +, 0)$ is a (finite) monoid and A acts on Q from the left. Then, each forest s over A evaluates to some element $s^{\mathbb{M}} \in Q$. A *forest language* (that is, a set of forests) L is *recognizable* in $\mathbb{M}$ if $L = \{s : s^{\mathbb{M}} \in F\}$ for some $F \subseteq Q$ and is *regular* if is recognizable in some finite forest automaton.

In this talk we define a class of future branching time temporal logics on forests similar to those in [1, 2], give a noneffective algebraic characterization when the set of modalities are each regular forest languages and give a polynomial-time decision algorithm for the definability problem of the EF* fragment of the variant of the logic CTL applicable for these forest languages, using methods similar to [3, 4].

The main result is the following:

Theorem 1 *A (regular) forest language is definable in* EF* *if and only if its minimal forest automaton satisfies the identities $p + q = q + p$ ad $aap = ap$ for each state p, q and symbol a, the implication $p \leq q \Rightarrow p + q = q$ and that $\leq$ is a partial order where $\leq$ is the preorder generated by $p \leq p + q$, $p \leq ap$.*

References

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