Two solutions for a singular elliptic problem indefinite in sign

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Singular problems arise in the study of non-Newtonian fluids, boundary layer phenomena for viscous fluids, chemical heterogeneous catalysts, as well as in the theory of heat conduction in electrically conducting materials. An increasing attention to singular stationary or evolution equations has been paid in the last decades.

In the present talk we deal with a singular elliptic problem involving a nonlinearity which is indefinite in sign:

$$\begin{cases} -\Delta u = (\lambda u^{s-1} - u^{r-1})\chi_{\{u>0\}} & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \qquad (P_\lambda)$$

where Ω is a bounded domain in $\mathbb{R}^N$ ($N \in \mathbb{N}$) with smooth boundary $\partial\Omega$, $0 < r \leq 1 < s < 2$, λ is a positive parameter and $\chi_{\{u>0\}}$ the characteristic function corresponding to the set $\{u > 0\}$.

We answer to a conjecture of Montenegro and Silva ([2]) about the existence of a positive solution for the above problem and prove the following:

Theorem 1 *There exists $\Lambda > 0$ such that problem (P_λ) has no positive solution for $\lambda < \Lambda$ and two distinct nontrivial nonnegative weak solutions for $\lambda > \Lambda$. One of them belongs to $\mathrm{int}(C_0^1(\overline{\Omega})_+)$ and corresponds to a local minimum point of the energy functional associated to problem (P_λ).*

We point out that the energy functional associated to (P_λ) is not Gâteaux differentiable in the Sobolev space $W_0^{1,2}(\Omega)$ and the classical critical point theory does not apply. In our proof truncation techniques and variational arguments are employed.

The present result has been jointly obtained with G. Anello (see [1]).

References

[1] G. Anello, F. Faraci, Two solutions for a singular elliptic problem indefinite in sign, *Nonlinear Differential Equations and Applications NoDEA* (2015), to appear.

[2] M. Montenegro, E. Silva, Two solutions for a singular elliptic equation by variational methods, *Ann. Sc. Norm. Super. Pisa Cl. Sci.* **11** (2012), 143–165.