Practical distance-constrained labelling of graphs

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Hale introduced graph colouring problems, inspired by frequency assignment [1]. This problem is a generalization of the map-colouring problem. At map colouring, we need to assign colours to the vertices of a graph in such a way, that no two adjacent vertices have the same colour. Here, a monotonic decreasing function is given $f : \{1, \ldots, k\} \to \mathbb{N}$, and we need to construct a colouring $c : V \to 1, \ldots, n$, such that if $d(u, v) \leq k$, then $|(c(u), c(v)| \geq f(d(u, v))$ where d is the edge-distance of nodes. A graph theoretical question looks to find the minimum number of colours for a given f and a class of graphs [2, 3].

If we have a concrete graph G, the problem is a combinatorial optimization problem, so we can use the usual methods to solve this individual program. In this paper, we examine the *min-conflicts* method of Artificial Intelligence [4], which performs well on map-colouring problems. If we fix the number of colours, the problem becomes a constraint satisfaction problem [5]. Therefore, we can use the tools of Constraint Logic Programming to solve this specific problem [6], as well as to determine the minimal n. We show several of the most interesting parts of our implementation, and the results of our benchmarks.

References

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