Equational properties of fixed point operations

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Most computer scientists are aware of the fact that just about every aspect of their study is related to fixed points. The importance of fixed points is due to the fact that the semantics of recursion and iteration is usually captured by fixed points of functions, functors, or other constructors. Fixed point operations have been widely used in automata and formal language theory and their generalizations, in the semantics of programming languages, abstract data types, process algebra and concurrency, rewriting, programming logics and verification, complexity theory, and many other fields.

A general study of the equational properties of fixed point operations has been carried out in the framework or *iteration theories*, or *iteration categories*, introduced in 1980 independently in [1] and [4].[1] Results on iteration theories obtained until the mid 1990's were summarized in [2] and [3]. For some more recent results see [5, 6, 7, 8, 9, 10]. It has been shown that the axioms of iteration theories properly capture the equational properties of several fixed point operations used in computer science, including the least fixed point operation over monotonic or continuous functions over complete lattices or directed complete partial orders, the unique fixed point operation on contractive functions over complete metric spaces, the initial fixed point operation over algebraically complete categories, and others. In the talk we review the axiomatization of iteration theories and provide several completeness results.

References

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[1] In [4], iteration theories were called "generalized iterative theories".